

Proposed solutions for tutorial 3

*Intermediate Microeconomics (UTS 23567)\**

Preliminary and incomplete

Available at <https://backwardinduction.blog/tutoring/>

Office hours on Mondays from 9 am till 10 am in building 8 on level 9

Please whatsapp me on 0457871540 so I could meet you at the door, I don't have an internal phone

Also please whatsapp if you have questions, I won't be able to answer through whatsapp but I will give answer during office hours or, since you are very unlikely to have ask a unique question, in the beginning of next tutorial

Sergey V. Alexeev

3 of April, 2018

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\*Questions for the tutorial were provided by Massimo Scotti, slide and textbook are by Nechyba (2016). Solutions and commentary are by Sergey Alexeev (e-mail: sergei.v.alexeev@gmail.com). (Btw this is a much better book Varian (1987))

### Question 1

- Suppose Mark likes both beer and wine
- Let  $x_1$  and  $x_2$  denote liters of beers and liters of wine respectively
- Assume that Mark's tastes over bundles of beer and wine are described by the following utility function:

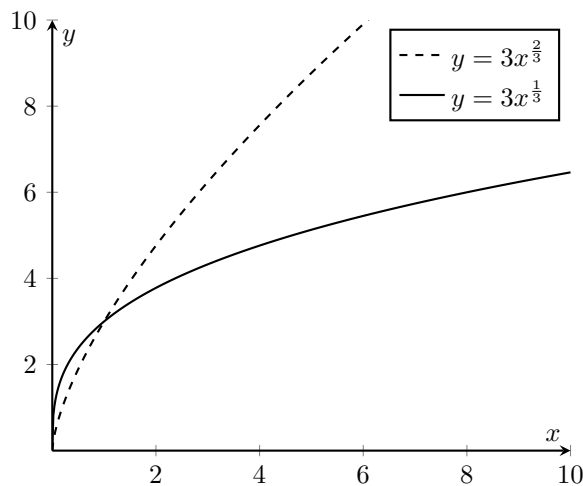
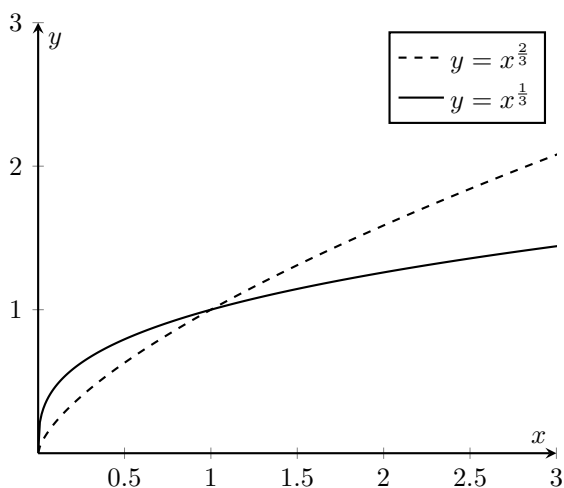
$$u(x_1, x_2) = 3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}$$

1.a.

Consider bundles  $A = (1, 27)$  and  $B = (27, 1)$

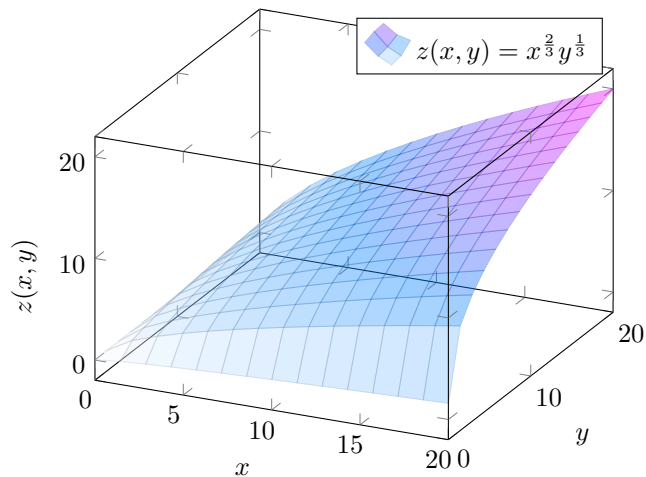
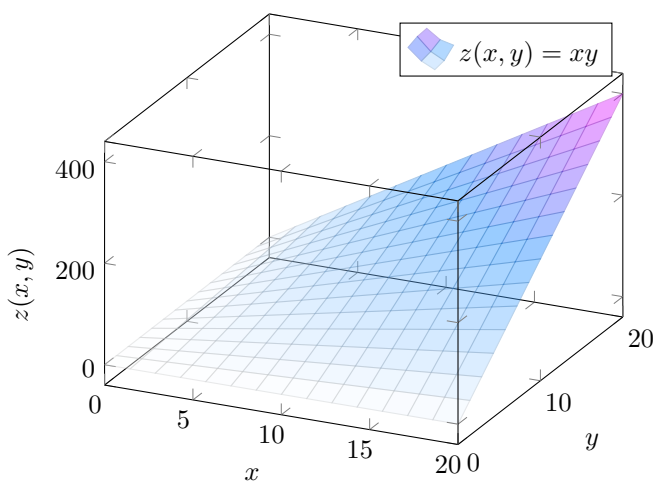
Which one does Mark prefer?

Before we start take a look at the function above and note



Before answering the question take a closer look at  $3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}$  and note that  $x^{\frac{2}{3}} > x^{\frac{1}{3}}$  for all  $x > 1$ .

A multiplication by number 3 does not change it, it simple scales numbers up



Clearly function  $z(x, y) = xy$  is maximized if  $x = y$  because if  $z(1, 0) = 1$ ,  $z(1, 1) = 1$ ,  $z(2, 1) = 2$ ,  $z(2, 2) = 4$  etc.

However,  $z(x, y) = x^{\frac{2}{3}}y^{\frac{1}{3}}$  is maximized if  $x > y$  note that  $z(2, 1) = 1.58$  but  $z(1, 2) = 1.25$

## Utility functions are mathematical functions with particular properties

Utility function is a way of assigning a number to every possible consumption bundle such that more-preferred bundles get assigned larger numbers than less-preferred bundles.

That is, a bundle  $(x_1, x_2)$  is preferred to a bundle  $(y_1, y_2)$  if utility of  $(x_1, x_2)$  is larger than the utility of  $(y_1, y_2)$ . In symbols

$$(x_1, x_2) \succ (y_1, y_2) \Leftrightarrow u(x_1, x_2) > u(y_1, y_2).$$

### Answer to 1.a

We need to use the function

$$u(x_1, x_2) = 3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}$$

to understand which bundle out is more preferred  $A = (1, 27)$  or  $B = (27, 1)$

Let's relabel bundles in a more informative way, e.g.

$$A = (1, 27) \equiv (1, 27)_A$$

this way we know the composition of a bundle and how it is designated in the question.

To see which bundle he likes better we need to evaluate the function at both points and compare the function's value

$$u(x_1, x_2) \Big|_{(1, 27)_A} = 3(1)^{\frac{2}{3}}(27)^{\frac{1}{3}} = 9$$

and

$$u(x_1, x_2) \Big|_{(27, 1)_B} = 3(27)^{\frac{2}{3}}(1)^{\frac{1}{3}} = 27$$

which means

$$\begin{aligned} u(x_1, x_2) \Big|_{(27, 1)_B} = 27 > u(x_1, x_2) \Big|_{(1, 27)_A} = 9 \\ \Leftrightarrow \\ (27, 1)_B \succ (1, 27)_A \end{aligned}$$

Question 1

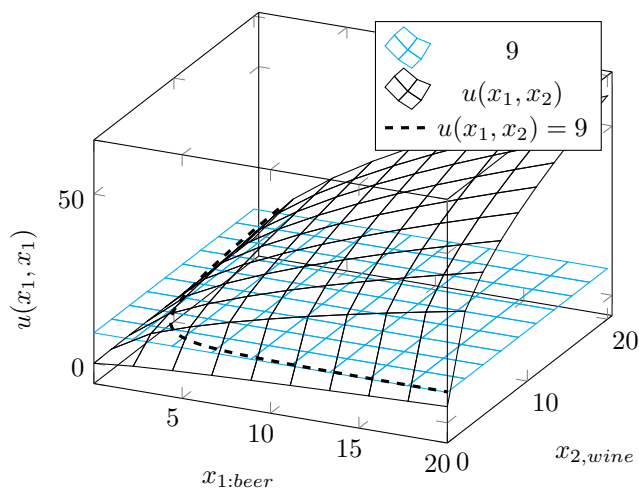
- Suppose Mark likes both beer and wine
- Let  $x_1$  and  $x_2$  denote liters of beers and liters of wine respectively
- Assume that Mark's tastes over bundles of beer and wine are described by the following utility function:

$$u(x_1, x_2) = 3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}$$

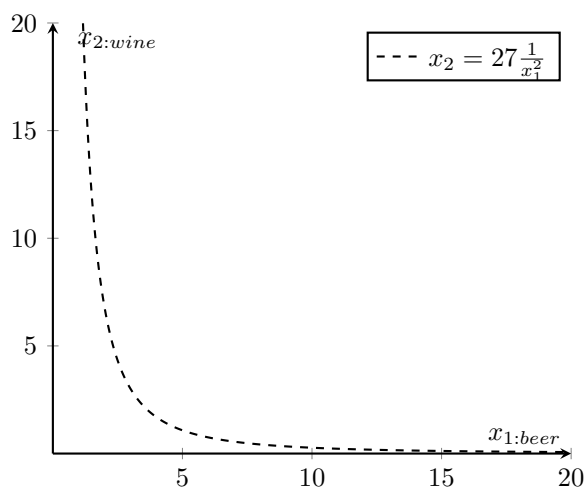
1.b.

Write the equation of the indifference curve on which bundle A lies?

Answer to 1.b



$$\begin{aligned} u(x_1, x_1) &= 9 \\ 3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}} &= 9 \\ x_1^{\frac{2}{3}}x_2^{\frac{1}{3}} &= 3 \\ x_2^{\frac{1}{3}} &= 3 \frac{1}{x_1^{\frac{2}{3}}} \\ x_2 &= 27 \frac{1}{x_1^2} \end{aligned}$$



Question 1

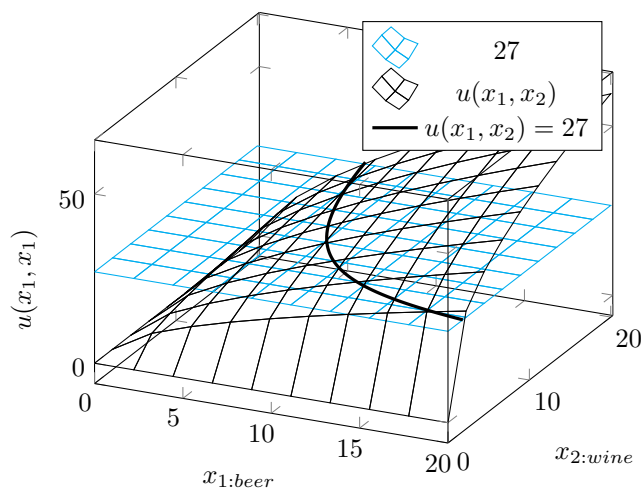
- Suppose Mark likes both beer and wine
- Let  $x_1$  and  $x_2$  denote liters of beers and liters of wine respectively
- Assume that Mark's tastes over bundles of beer and wine are described by the following utility function:

$$u(x_1, x_2) = 3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}$$

1.c.

Write the equation of the indifference curve on which bundle  $B$  lies?

Answer to 1.c



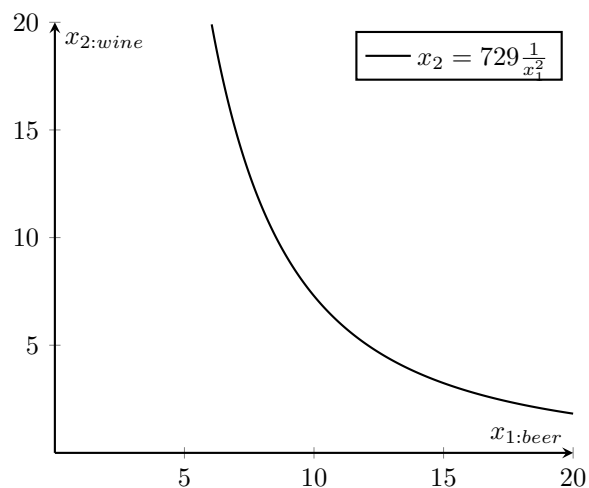
$$u(x_1, x_1) = 27$$

$$3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}} = 27$$

$$x_1^{\frac{2}{3}}x_2^{\frac{1}{3}} = 9$$

$$x_2^{\frac{1}{3}} = 9 \frac{1}{x_1^{\frac{2}{3}}}$$

$$x_2 = 729 \frac{1}{x_1^2}$$



### Question 1

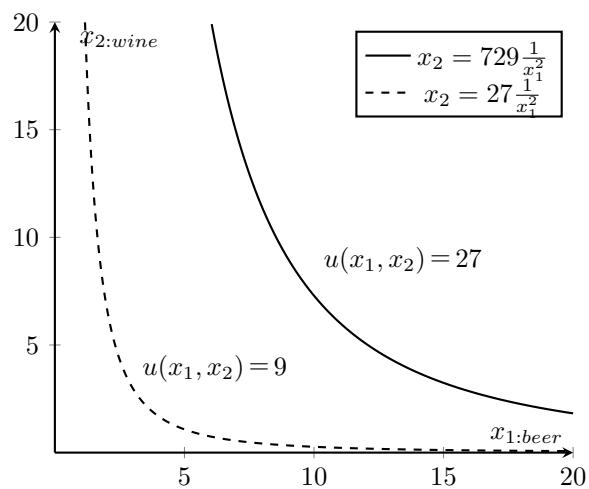
- Suppose Mark likes both beer and wine
- Let  $x_1$  and  $x_2$  denote liters of beers and liters of wine respectively
- Assume that Mark's tastes over bundles of beer and wine are described by the following utility function:

$$u(x_1, x_2) = 3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}$$

1.d.

In a graph where you put liters of beer on the horizontal axis and liters of wine on the vertical axes, represent the two indifference curves.

Answer to 1.d



## Question 1

- Suppose Mark likes both beer and wine
- Let  $x_1$  and  $x_2$  denote liters of beers and liters of wine respectively
- Assume that Mark's tastes over bundles of beer and wine are described by the following utility function:

$$u(x_1, x_2) = 3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}$$

**1.e.**

Find the value of the MRS at bundle  $A$  and at bundle  $B$ ?

Before we start recall the definitions

$$MRS(x_1, x_2) = -\frac{\partial u(x_1, \bar{x}_2)/\partial x_1}{\partial u(\bar{x}_1, x_2)/\partial x_2}$$

where  $\bar{x}_1$  indicates that  $x_1$  is fixed (i.e.  $x_1 = \bar{x}_1 \in \mathbb{R}_{\geq 0}$ ). Same for  $\bar{x}_2$

Let's expand the definition of MRS a little further to see what is hidden behind those symbols

The numerator of MRS is a marginal utility of  $x_1$  something that does not on its own have a behavioral interpretation

$$\frac{\partial u(x_1, \bar{x}_2)}{\partial x_1} = \underbrace{u_{x_1}(x_1, x_2)}_{\text{proper math notation}} = \underbrace{MU_1(x_1, x_2)}_{\text{textbook notation}} = \underbrace{\lim_{\Delta x_1 \rightarrow 0} \frac{u(x_1 + \Delta x_1, \bar{x}_2) - u(x_1, \bar{x}_2)}{\Delta x_1}}_{\text{definition}}$$

and the denominator is marginal utility  $x_2$  (same definition as above with 1 and 2 interchanged).

Then

$$MRS(x_1, x_2) = -\frac{\lim_{\Delta x_1 \rightarrow 0} \frac{u(x_1 + \Delta x_1, \bar{x}_2) - u(x_1, \bar{x}_2)}{\Delta x_1}}{\lim_{\Delta x_2 \rightarrow 0} \frac{u(\bar{x}_1, x_2 + \Delta x_2) - u(\bar{x}_1, x_2)}{\Delta x_2}} = -\frac{\lim_{\Delta x_1 \rightarrow 0} (\text{slope of function } u(x_1, \bar{x}_2))}{\lim_{\Delta x_2 \rightarrow 0} (\text{slope of function } u(\bar{x}_1, x_2))}$$

but what is the meaning of  $\lim_{\Delta x_1 \rightarrow 0}$  ?

Let us understand it on an example.

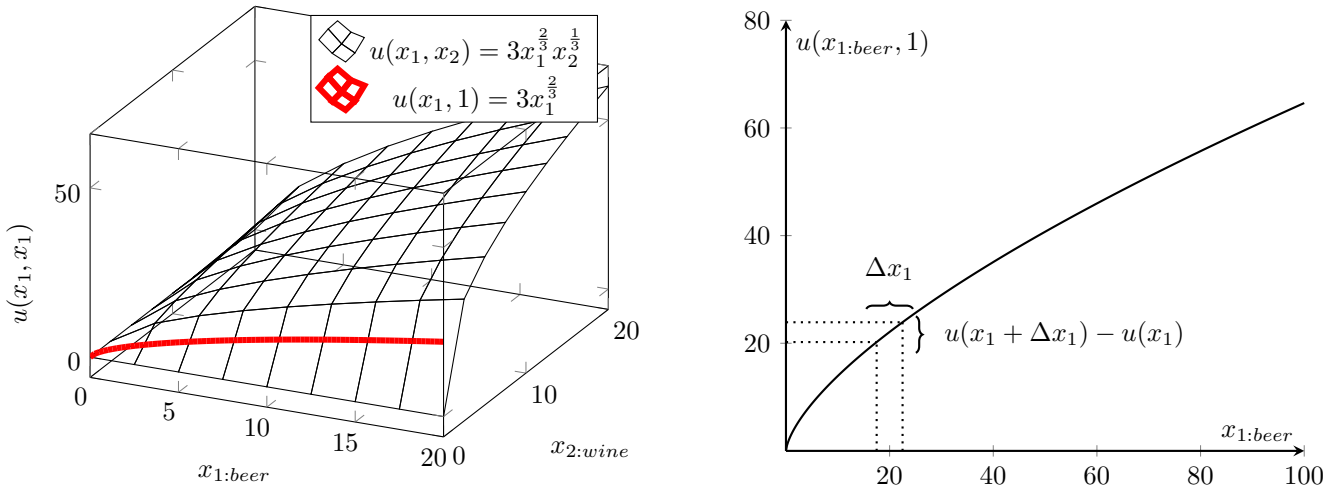
Take the function

$$u(x_1, x_2) = 3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}$$

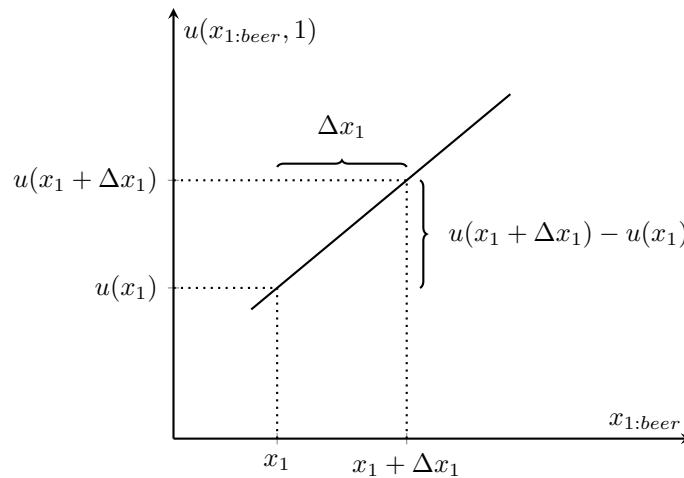
and fix  $x_2$  by evaluating the function at the point

$$u(x_1, x_2) \Big|_{(x_1, 1)} = 3x_1^{\frac{2}{3}}$$

clearly the function now become univariate and we can plot it



unfortunately this curve does not have a slope, however if we look close enough we can get one because any curve is a line if one looks close enough



Essentially

$$\begin{aligned} MRS(x_1, x_2) &= -\frac{\partial u(x_1, \bar{x}_2)/\partial x_1}{\partial u(\bar{x}_1, x_2)/\partial x_2} \\ &= -\frac{\lim_{\Delta x_1 \rightarrow 0} \frac{u(x_1 + \Delta x_1, \bar{x}_2) - u(x_1, \bar{x}_2)}{\Delta x_1}}{\lim_{\Delta x_2 \rightarrow 0} \frac{u(\bar{x}_1, x_2 + \Delta x_2) - u(\bar{x}_1, x_2)}{\Delta x_2}} \\ &= -\frac{\lim_{\Delta x_1 \rightarrow 0} (\text{slope of function } u(x_1, \bar{x}_2))}{\lim_{\Delta x_2 \rightarrow 0} (\text{slope of function } u(\bar{x}_1, x_2))} \\ &= -\frac{\text{“local value” of } x_{1:beer}}{\text{“local value” of } x_{2:wine}} \end{aligned}$$

which could be understood in words as the following:



- A value of  $x_1$  normalized by value of  $x_2$
- A value  $x_1$  in units of value of  $x_2$
- If  $x_2$  were money then it says how much you are ok (to stay put) to pay for the  $x_1$
- If  $x_2$  is your nominal wealth, then dividing by a normalization factor  $x_1$  it becomes real wealth (kinda like nominal money normalized by inflation, it's fairly loose analogy, but might help someone)
- A rate at which a consumer is willing (in a sense that he won't be worse off) to substitute (exchange, trade)  $x_2$  for  $x_1$

### Answer to 1.e

This function

$$u(x_1, x_2) = 3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}$$

has following marginal utilities

$$\frac{\partial u(x_1, x_2)}{\partial x_1} = 2x_1^{-\frac{1}{3}}x_2^{\frac{1}{3}}$$

$$\frac{\partial u(x_1, x_2)}{\partial x_2} = x_1^{\frac{2}{3}}x_2^{-\frac{2}{3}}$$

which gives us MRS

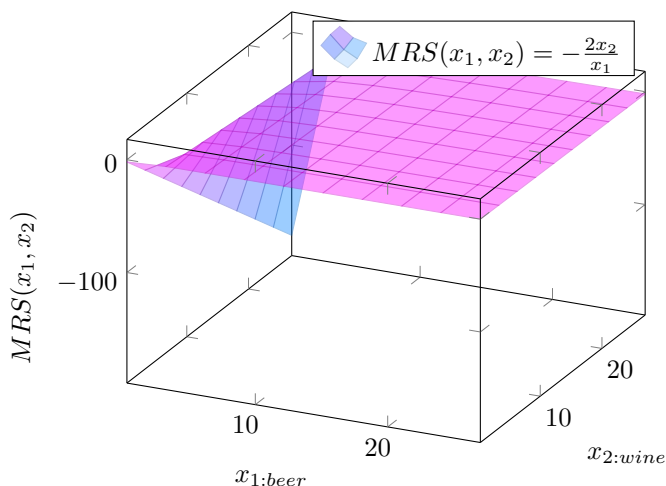
$$MRS(x_1, x_2) = -\frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} = -\frac{2x_1^{-\frac{1}{3}}x_2^{\frac{1}{3}}}{x_1^{\frac{2}{3}}x_2^{-\frac{2}{3}}} = -\frac{2x_2:wine}{x_1:beer}$$

Evaluating it two bundles gives the following

$$MRS(x_1, x_2) \Big|_{(1,27)_A} = -\frac{2(27)}{1} = -54$$

$$MRS(x_1, x_2) \Big|_{(27,1)_B} = -\frac{2(1)}{27} = -\frac{2}{27} \approx -0.074$$

He hates  $(1, 27)_A$  so much that willing to give away 54 litres of wine for an extra liter of beer, whereas with bundle  $(27, 1)_B$  in hands he is willing to give away only 0.07 litres of beer for an extra litre of wine. It could be verified with the figure



## Another way to understand MRS

Why MRS is it defined the way it is defined and why is there a minus?

The definition of MRS has to contain information on a desire of a consumer to maximize utility. That is he chooses the willingness to trade to make sure that he is not worse off.

This idea can be easily expressed by marginal utilities

$$du(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_1} dx_1 + \frac{\partial u(x_1, x_2)}{\partial x_2} dx_2.$$

This relationship holds for any function, including the utility function.

Then to say that consumer chooses to trade to guarantee that he is not getting worse is identical to the following equation

$$du(x_1, x_2) = \frac{\partial u(x_1, x_2)}{\partial x_1} dx_1 + \frac{\partial u(x_1, x_2)}{\partial x_2} dx_2 \stackrel{set}{=} 0$$

which will give as a change  $(dx_1, dx_2)$  that keeps utility constant

$$\frac{\partial u(x_1, x_2)}{\partial x_1} dx_1 + \frac{\partial u(x_1, x_2)}{\partial x_2} dx_2 \stackrel{set}{=} 0 \Rightarrow \frac{dx_2}{dx_1} = -\frac{\partial u(x_1, \bar{x}_2)/\partial x_1}{\partial u(x_1, \bar{x}_2)/\partial x_2}$$

Alternatively we can think of the indifference curve as being described by a function  $x_2(x_1)$ , the following needs to be satisfied by construction (by definition of indifference curve):

$$u(x_1, x_2(x_1)) \equiv k$$

then differentiating both sides with respect to  $x_1$  gives

$$\frac{\partial u(x_1, x_2)}{\partial x_1} + \frac{\partial u(x_1, x_2)}{\partial x_2} \frac{\partial x_2(x_1)}{\partial x_1} = 0$$

which can be rearranged into

$$\frac{\partial x_2(x_1)}{\partial x_1} = -\frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2}$$

### Question 1

- Suppose Mark likes both beer and wine
- Let  $x_1$  and  $x_2$  denote liters of beers and liters of wine respectively
- Assume that Mark's tastes over bundles of beer and wine are described by the following utility function:

$$u(x_1, x_2) = 3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}$$

**1.f.**

Are Mark's tastes characterized by a diminishing marginal rate of substitution? Motivate your answer

### Answer to 1.f

$$MRS(x_1, x_2) = -\frac{2x_{2:wine}}{x_{1:beer}}$$

It is decreasing in  $x_1$

His willingness to get more beer for wine gets smaller, because he is starting to hate beer when he has tons of it.

## Question 1 Checklist

- Suppose Mark likes both beer and wine
- Let  $x_1$  and  $x_2$  denote liters of beers and liters of wine respectively
- Assume that Mark's tastes over bundles of beer and wine are described by the following utility function:

$$u(x_1, x_2) = 3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}$$

### 1.a

Consider bundles  $A = (1, 27)$  and  $B = (27, 1)$

### 1.b

Write the equation of the indifference curve on which bundle  $A$  lies?

### 1.c

Write the equation of the indifference curve on which bundle  $B$  lies?

### 1.d

In a graph where you put liters of beer on the horizontal axis and liters of wine on the vertical axes, represent the two indifference curves.

### 1.e

Find the value of the MRS at bundle  $A$  and at bundle  $B$

### 1.f

Are Mark's tastes characterized by a diminishing marginal rate of substitution? Motivate your answer

## Question 2

Answer parts a-f in question 2 but for consumer John whose tastes over bundles of beer and wine are described by the following utility function:

$$u(x_1, x_2) = x_1 + x_2$$

### 2.a

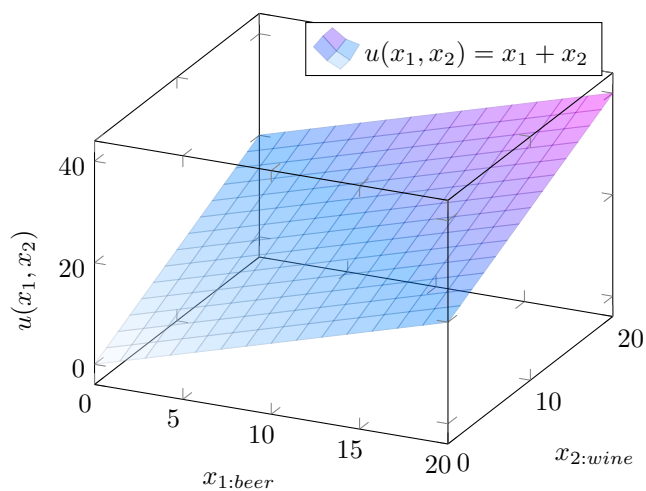
Consider bundles  $A = (1, 27)$  and  $B = (27, 1)$

Compare which one does Mark and John prefer?

### Answer to 2.a

Direct comparison gives

$$\begin{aligned} u(x_1, x_2) \Big|_{(1,27)_A} &= 1 + 27 = 28 \\ u(x_1, x_2) \Big|_{(27,1)_B} &= 1 + 27 = 28 \\ u(x_1, x_2) \Big|_{(1,27)_A} &= u(x_1, x_2) \Big|_{(27,1)_B} \Leftrightarrow (1, 27)_A \sim (27, 1)_B \end{aligned}$$



### Question 2

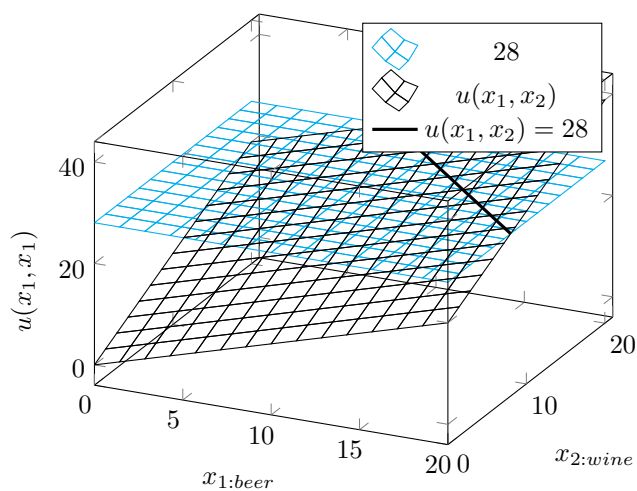
Answer parts a-f in question 2 but for consumer John whose tastes over bundles of beer and wine are described by the following utility function:

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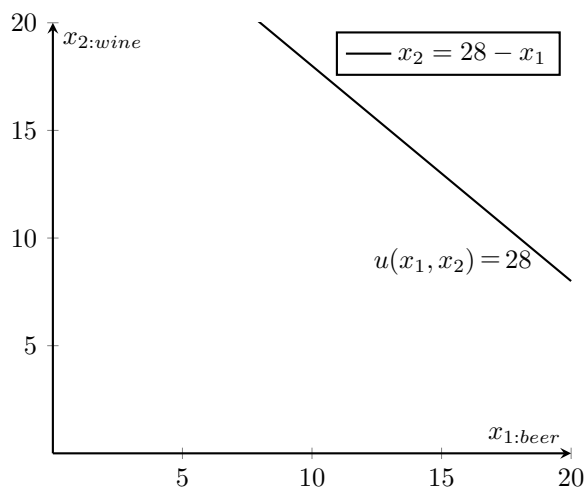
#### 2.b

Write the equation of the indifference curve on which bundle A lies?

#### Answer to 2.b



$$\begin{aligned}u(x_1, x_2) &= 28 \\x_1 + x_2 &= 28 \\x_2 &= 28 - x_1\end{aligned}$$



### Question 2

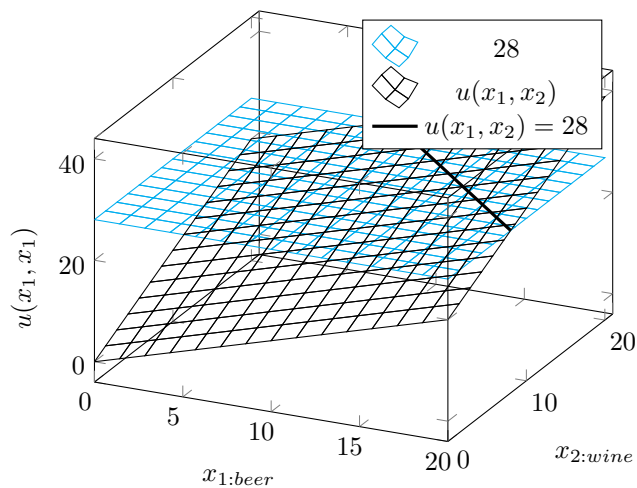
Answer parts a-f in question 2 but for consumer John whose tastes over bundles of beer and wine are described by the following utility function:

$$u(x_1, x_2) = x_1 + x_2$$

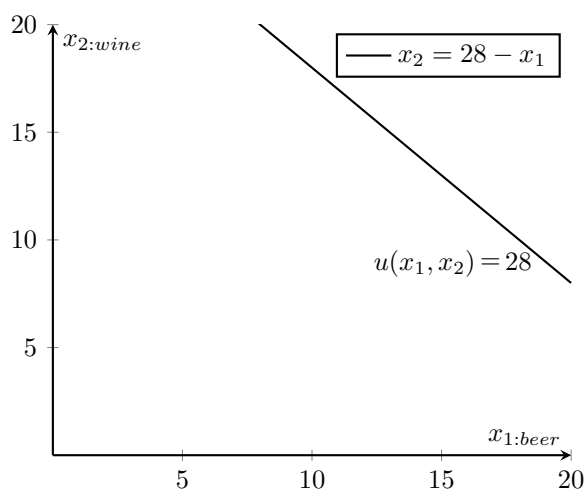
**2.c**

Write the equation of the indifference curve on which bundle *B* lies?

**Answer to 2.c**



$$\begin{aligned} u(x_1, x_2) &= 28 \\ x_1 + x_2 &= 28 \\ x_2 &= 28 - x_1 \end{aligned}$$



## Question 2

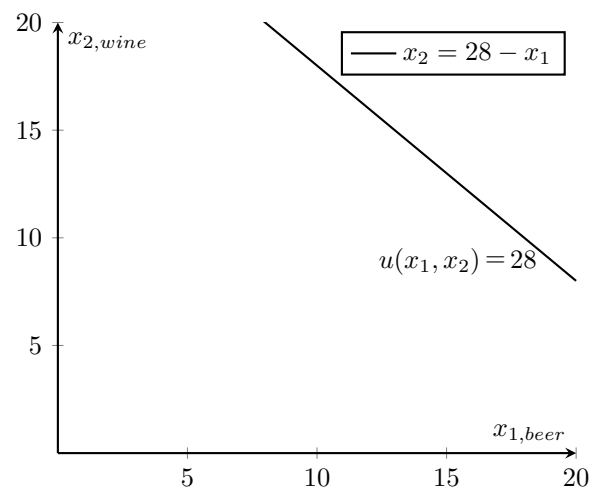
Answer parts a-f in question 2 but for consumer John whose tastes over bundles of beer and wine are described by the following utility function:

$$u(x_1, x_2) = x_1 + x_2$$

### 2.d

In a graph where you put liters of beer on the horizontal axis and liters of wine on the vertical axes, represent the two indifference curves.

### Answer to 2.d





## Question 2

Answer parts a-f in question 2 but for consumer John whose tastes over bundles of beer and wine are described by the following utility function:

$$u(x_1, x_2) = x_1 + x_2$$

**2.e**

Find the value of the MRS at bundle  $A$  and at bundle  $B$ ?

### Answer to 2.e

Applying MRS here is an overkill, since clearly

$$\frac{\partial u(x_1, x_2)}{\partial x_1} = 1$$

$$\frac{\partial u(x_1, x_2)}{\partial x_2} = 1$$

and

$$MRS(x_1, x_2) = -\frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} = -\frac{1}{1} = -1$$

which is constant and does not depend on location of  $(x_1, x_2)$

In general note that if marginal utilities are constant MRS collapses into a slope. In this sense MRS is generalization of a slope. Note:

$$\begin{aligned} MRS(x_1, x_2) &= -\frac{\partial u(x_1, \bar{x}_2)/\partial x_1}{\partial u(\bar{x}_1, x_2)/\partial x_2} \\ &= -\frac{\lim_{\Delta x_1 \rightarrow 0} \frac{u(x_1 + \Delta x_1, \bar{x}_2) - u(x_1, \bar{x}_2)}{\Delta x_1}}{\lim_{\Delta x_2 \rightarrow 0} \frac{u(\bar{x}_1, x_2 + \Delta x_2) - u(\bar{x}_1, x_2)}{\Delta x_2}} \\ &= -\frac{\frac{u(x_1 + \Delta x_1, \bar{x}_2) - u(x_1, \bar{x}_2)}{\Delta x_1}}{\frac{u(\bar{x}_1, x_2 + \Delta x_2) - u(\bar{x}_1, x_2)}{\Delta x_2}} \\ &= -\underbrace{\left(\frac{\Delta x_2}{\Delta x_1}\right)}_{\text{a slope in } (x_2, x_1) \text{ plane}} \left(\frac{u(x_1 + \Delta x_1, \bar{x}_2) - u(x_1, \bar{x}_2)}{u(\bar{x}_1, x_2 + \Delta x_2) - u(\bar{x}_1, x_2)}\right) \end{aligned}$$

where second line is by definition of MRS, third because limit of a constant is that constant itself, the fourth is just a rearrangement

## Question 2

Answer parts a-f in question 2 but for consumer John whose tastes over bundles of beer and wine are described by the following utility function:

$$u(x_1, x_2) = x_1 + x_2$$

### 2.f

Are John's tastes characterized by a diminishing marginal rate of substitution? Motivate your answer.

### Answer to 2.f

nope

## Question 2 Checklist

Answer parts a-f in question 2 but for consumer John whose tastes over bundles of beer and wine are described by the following utility function:

$$u^m(x_1, x_2) = 3x_1^{\frac{2}{3}}x_2^{\frac{1}{3}}$$

$$u^j(x_1, x_2) = x_1 + x_2$$

### 2.a

Consider bundles  $A = (1, 27)$  and  $B = (27, 1)$

Compare which one does Mark and John prefer?

### 2.b

Write the equation of the indifference curve on which bundle  $A$  lies?

Compare curves for Mark and John?

### 2.c

Write the equation of the indifference curve on which bundle  $B$  lies?

Compare curves for Mark and John?

### 2.d

In a graph where you put liters of beer on the horizontal axis and liters of wine on the vertical axes, represent the two indifference curves.

Compare curves for Mark and John?

### 2.e

Find the value of the MRS at bundle  $A$  and at bundle  $B$ ?

Compare those for Mark and John?

### 2.f

Compare Mark's and John's diminishing marginal rates of substitution?

### Question 3

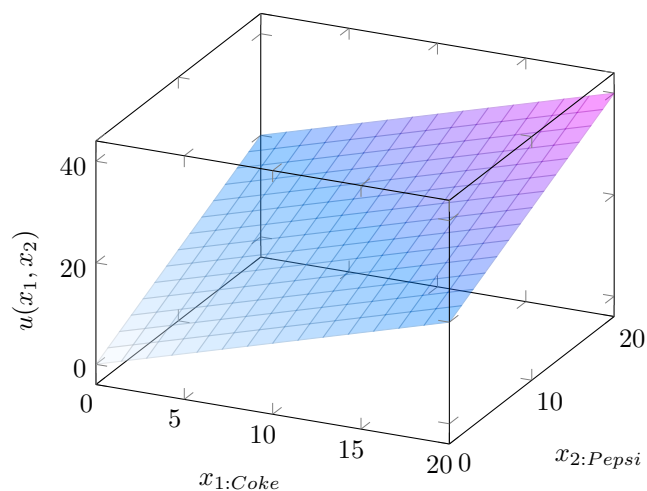
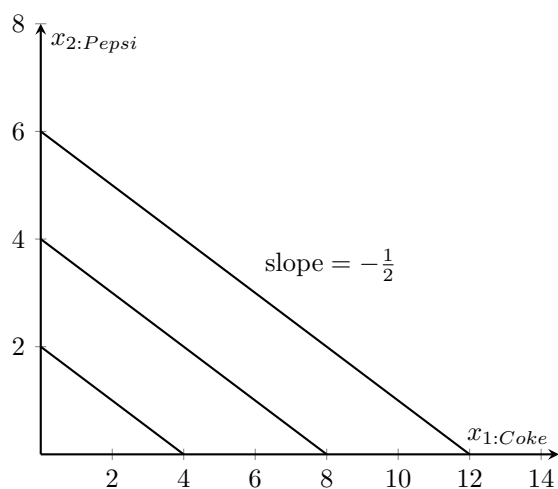
- Suppose that for a consumer Coke and Pepsi taste exactly the same
- Suppose also that Coca-Cola makes the marketing decision to sell only cans of Coke that are half the size of cans of Pepsi
- With this decision, cans of Coke contain 167.5 ml while cans of Pepsi contain 335ml

#### 3.a

In a graph where you put cans of Coke on the horizontal axis and cans of Pepsi on the vertical axes, draw a map of indifference curves for this consumer.

#### Answer to 3.a

They are substitutes



### Question 3

- Suppose that for a consumer Coke and Pepsi taste exactly the same
- Suppose also that Coca-Cola makes the marketing decision to sell only cans of Coke that are half the size of cans of Pepsi
- With this decision, cans of Coke contain 167.5 ml while cans of Pepsi contain 335ml

#### 3.b

What is the MRS of cans of Pepsi for cans of Coke?

#### Answer to 3.b

The definition of MRS

$$MRS(x_1, x_2) = -\frac{\partial u(x_1, x_2)/\partial x_{1:Coke}}{\partial u(x_1, x_2)/\partial x_{2:Pepsi}}$$

which can be verbally translated as

- A value of  $x_{1:Coke}$  normalized by value of  $x_{2:Pepsi}$
- A value  $x_{1:Coke}$  in units of value of  $x_{2:Pepsi}$
- If  $x_{2:Pepsi}$  were money then it says how much you are ok (to stay put) to pay for the  $x_{1:Coke}$
- A rate at which a consumer is willing (in a sense that he won't be worse off) to substitute (exchange, trade)  $x_{2:Pepsi}$  for  $x_{1:Coke}$

Because 1 unit of Pepsi worth two 2 units of Coke and they are substitutes then

$$MRS = -\frac{1}{2}$$

again note that in this case MRS and the slope coincide.

### Question 3

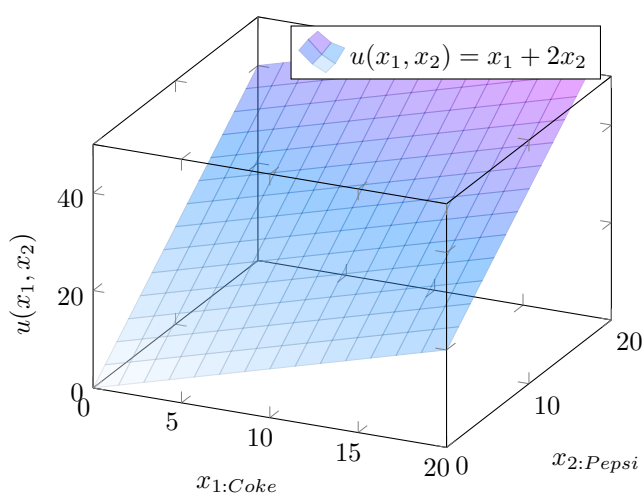
- Suppose that for a consumer Coke and Pepsi taste exactly the same
- Suppose also that Coca-Cola makes the marketing decision to sell only cans of Coke that are half the size of cans of Pepsi
- With this decision, cans of Coke contain 167.5 ml while cans of Pepsi contain 335ml

#### 3.c

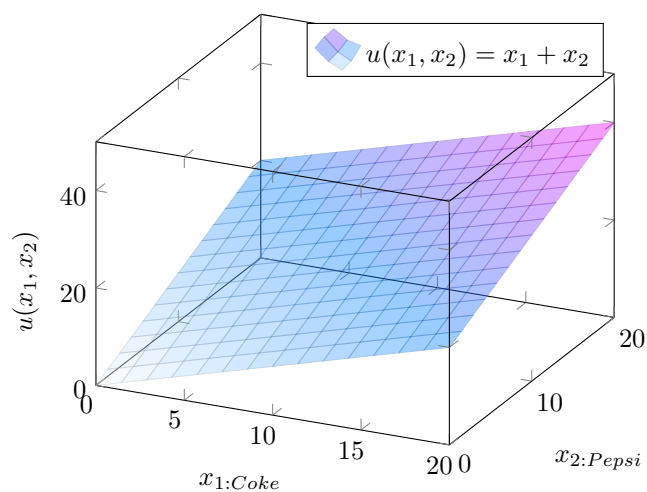
- What utility function represents the case considered in the this question
- Let  $x_1$  denote the amount of Coke and  $x_2$  the amount of Pepsi?

#### Answer to 3.c

$$u(x_1, x_2) = x_1 + 2x_2$$



Coefficient 2 tilts the utility function toward  $x_2$



For comparison

### Question 3 Checklist

- Suppose that for a consumer Coke and Pepsi taste exactly the same
- Suppose also that Coca-Cola makes the marketing decision to sell only cans of Coke that are half the size of cans of Pepsi
- With this decision, cans of Coke contain 167.5 ml while cans of Pepsi contain 335ml

#### 3.a

In a graph where you put cans of Coke on the horizontal axis and cans of Pepsi on the vertical axes, draw a map of indifference curves for this consumer

#### 3.b

What is the MRS of cans of Pepsi for cans of Coke?

#### 3.c

- What utility function represents the case considered in the this question
- Let  $x_1$  denote the amount of Coke and  $x_2$  the amount of Pepsi?

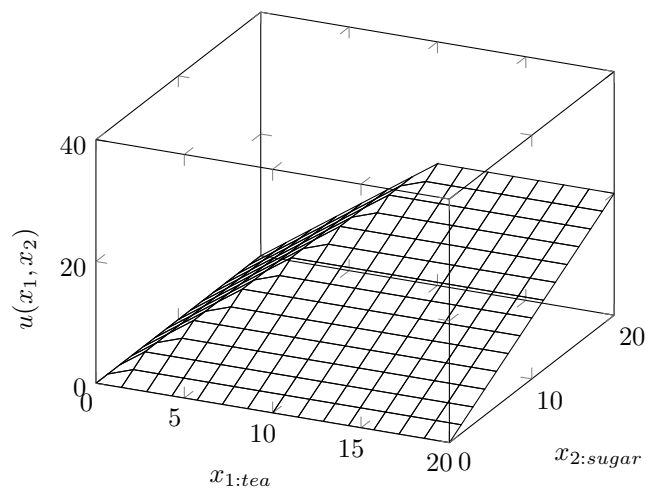
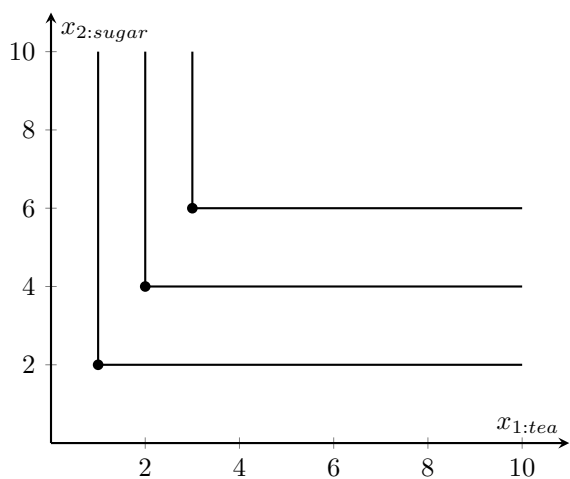
### Question 4

Suppose that a consumer always consumes packs of sugar and glasses of tea in the following fixed proportion: 2 packs of sugar for each glass of tea.

**4.a.**

In a graph where you put glasses of tea on the horizontal axis and packs of sugar on the vertical axes, draw a map of indifference curves for this consumer.

### Answer to 4.a



Even if you have  $x_1 = 6$ , but  $x_2 = 2$  you still at point  $(2, 2)$ , one good has no value without the other.

Note a left-side tilt toward  $x_2:sugar$



#### Question 4

Suppose that a consumer always consumes packs of sugar and glasses of tea in the following fixed proportion:  
2 packs of sugar for each glass of tea.

#### 4.b

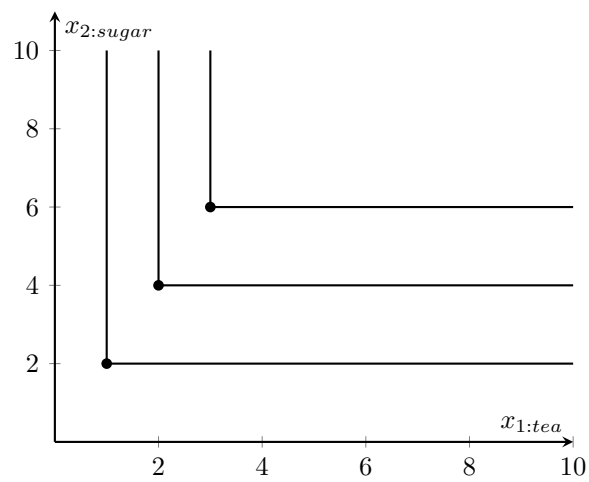
Is this consumer willing to substitute sugar for tea?

#### Answer to 4.b

Optimal points can not be characterized with slopes or tangents (which are slopes in a small neighborhood).

$$MRS(x_1, x_2) = -\frac{\partial u(x_1, \bar{x}_2)/\partial x_1}{\partial u(\bar{x}_1, x_2)/\partial x_2} \neq$$

But we still can reason verbally that consumer will never exchange one one another, because he/she needs both.



### Question 4

Suppose that a consumer always consumes packs of sugar and glasses of tea in the following fixed proportion:  
2 packs of sugar for each glass of tea.

**4.c**

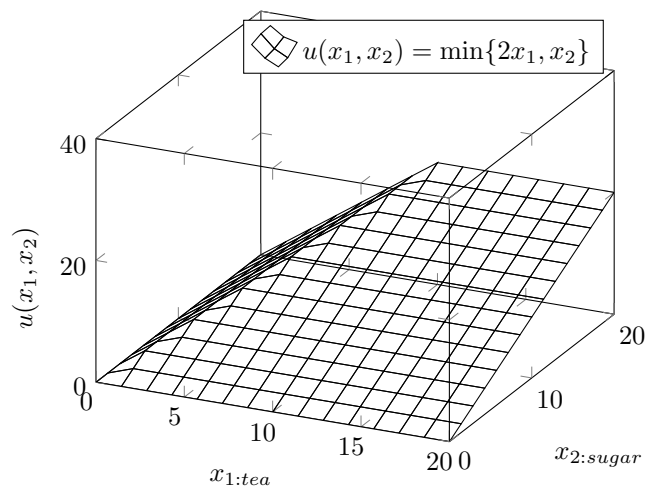
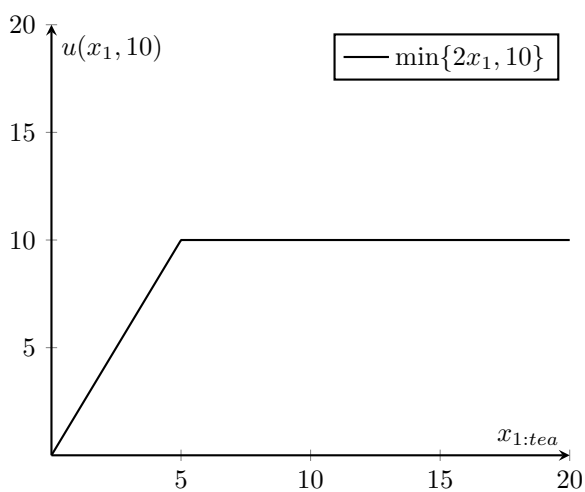
Make sense of the fact that the tastes of this consumer can be described by utility function  $u(x_1, x_2) = \min\{2x_1, x_2\}$ .

### Answer to 4.c

Expanding the function with minimum gives the following

$$u(x_1, x_2) = \min\{2x_1, x_2\} = \begin{cases} 2x_1 & \text{if } 2x_1 < x_2 \\ x_2 & \text{if } x_2 < 2x_1 \end{cases}$$

with the following graphical interpretation



#### Question 4 Checklist

Suppose that a consumer always consumes packs of sugar and glasses of tea in the following fixed proportion:  
2 packs of sugar for each glass of tea.

**4.a**

In a graph where you put glasses of tea on the horizontal axis and packs of sugar on the vertical axes, draw a map of indifference curves for this consumer

**4.b**

Is this consumer willing to substitute sugar for tea?

**4.c**

Make sense of the fact that the tastes of this consumer can be described by utility function  $u(x_1, x_2) = \min\{2x_1, x_2\}$

Question 5

5.a

Do the following two utility functions represent the same tastes?

$$u^a(x_1, x_2) = x_1^2 x_2^2$$

$$u^b(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

Answer to 5.a

$$\frac{\partial u^a(x_1, x_2)}{\partial x_1} = 2x_1 x_2^2$$

$$\frac{\partial u^a(x_1, x_2)}{\partial x_2} = 2x_1^2 x_2$$

$$MRS^a(x_1, x_2) = -\frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} = -\frac{2x_1 x_2^2}{2x_1^2 x_2} = -\frac{x_2}{x_1}$$

$$\frac{\partial u^b(x_1, x_2)}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$\frac{\partial u^b(x_1, x_2)}{\partial x_2} = \frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}$$

$$MRS^b(x_1, x_2) = -\frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} = -\frac{\frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}}{\frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}} = -\frac{x_2}{x_1}$$

$$MRS^a = MRS^b \Leftrightarrow \zeta^a = \zeta^b$$

Also it is clear that

$$x_1^2 x_2^2 \equiv \left(x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}\right)^4$$

## Question 5

## 5.b

Do the following two utility functions represent the same tastes?

$$u^a(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$$

$$u^b(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

## Answer to 5.b

$$\frac{\partial u^a(x_1, x_2)}{\partial x_1} = \frac{2}{3} x_1^{-\frac{1}{3}} x_2^{\frac{1}{3}}$$

$$\frac{\partial u^a(x_1, x_2)}{\partial x_2} = \frac{1}{3} x_1^{\frac{2}{3}} x_2^{-\frac{2}{3}}$$

$$MRS^a(x_1, x_2) = -\frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} = -\frac{\frac{2}{3} x_1^{-\frac{1}{3}} x_2^{\frac{1}{3}}}{\frac{1}{3} x_1^{\frac{2}{3}} x_2^{-\frac{2}{3}}} = -\frac{2x_2}{x_1}$$

$$\frac{\partial u^b(x_1, x_2)}{\partial x_1} = \frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}$$

$$\frac{\partial u^b(x_1, x_2)}{\partial x_2} = \frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}$$

$$MRS^b(x_1, x_2) = -\frac{\partial u(x_1, x_2)/\partial x_1}{\partial u(x_1, x_2)/\partial x_2} = -\frac{\frac{1}{2} x_1^{-\frac{1}{2}} x_2^{\frac{1}{2}}}{\frac{1}{2} x_1^{\frac{1}{2}} x_2^{-\frac{1}{2}}} = -\frac{x_2}{x_1}$$

$$MRS^a \neq MRS^b \Leftrightarrow \succsim^a \neq \succsim^b$$

## Question 5 Checklist

### 5.a

Do the following two utility functions represent the same tastes?

$$u^a(x_1, x_2) = x_1^2 x_2^2$$

$$u^b(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

### 5.b

Do the following two utility functions represent the same tastes?

$$u^a(x_1, x_2) = x_1^{\frac{2}{3}} x_2^{\frac{1}{3}}$$

$$u^b(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

## References

Nechyba, Thomas (2016). *Microeconomics: an intuitive approach with calculus*. Nelson Education.

URL: [https://sergeyvalexeev.files.wordpress.com/2018/03/inechyba\\_t\\_microeconomics\\_an\\_intuitive\\_approach\\_with\\_calculus.pdf](https://sergeyvalexeev.files.wordpress.com/2018/03/inechyba_t_microeconomics_an_intuitive_approach_with_calculus.pdf).

Varian, Hal R (1987). *Intermediate Microeconomics; a modern approach*.

URL: <https://sergeyvalexeev.files.wordpress.com/%202018/04/varian-intermediate-microeconomics.pdf>.