The Effect of the Test-Preparation Industry on Centralized College Admission*

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July 6, 2019

Abstract

This is the first paper that incorporates test preparation activities into college admission game theoretical models, and studies the incentives of college applicants, their test taking behavior, and the quality of sorting within this environment. Using an auction theoretical model, this paper shows that the fee-based commercial test preparation industry makes marks less informative about the underlying skills of applicants. An applicant with more fortunate opportunities to access test specific knowledge finds it rational to invest more into test preparation schools (regardless of the productivity of the knowledge) in order to increase his/her chances against less fortunate applicants who cannot access test specific knowledge. This strategic interaction complicates sorting under the centralized college admission system in general, and, more markedly, for highly competitive colleges or specializations.

Keywords: Auctions; Higher Education.
JEL codes: D44; I23.

*I thank Prof. Isa Hafalir and Dr. Kentaro Tomoeda for their help with this study.

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1 Introduction

Following the seminal work of Roth and Xing (1997), which delineates and solves the problem of congestion (a failure by participants to make sufficient offers and acceptances to clear the market), centralized matching has assumed a prominent place in economic theory and practice. Centralized matching has been successfully applied to medical residencies and public school choices, as an example. College admission represents a similar market and is known to suffer from a lack of coordination. It is widely believed that improved coordination by centralized matching would result in welfare benefits; however, the admission process in many countries still operates in a similar manner to those in decentralized labor markets. A better understanding of the reluctance to adopt a centralized college admission (CCA) system is a critical aspect of market design. This paper explores one possible explanation for this reluctance.

Unlike in the medical residency context (in which centralized matching begins after applicants and hospitals interview one another in a decentralized fashion), CCA uses standardized examination marks as a proxy for applicants' skills and colleges are typically assumed to prefer applicants with high marks to applicants with low marks. CCA has been studied extensively in economic literature; however, to date, no papers appear to have acknowledged that standardized examination marks are subject to what Haladyna et al. (1991) and Mehrens and Kaminski (1989) have dubbed "test marks pollution" (i.e., the skills-irrelevant test marks variance that is induced by commercial materials that have been specifically designed to improve test performance).

To better understand this idea, this paper studies the most recent wave of the introduction of CCA in ex-Soviet republics.[1] Unlike the applicants of many countries that embraced large-scale test standardization decades ago, the applicants in the ex-Soviet republics, due to their specific shared past, were admitted based on college and often department specific admission examinations (the pure antipode to CCA). Thus, this wave of CCA introductions in ex-Soviet republics where, previously, nothing that remotely resembled such a system existed, can be used to undertake a rare and useful quasi-experiment. In all of the cases studied (Gorgodze 2007; Prakhov and Yudkevich 2017), over time, the introduction of CCA in ex-Soviet republics created a private test-preparation (TP) industry that gradually redistributed the educational opportunities from poorer to wealthier households. Consequently, many colleges began to demonstrate a reluctance to admit students based on their centralized examination

[1] Years of implementation: Azerbaijan 1992, Kazakhstan 1999–2004, Russia 2001–2009, Kyrgyzstan 2002, the Ukraine 2004–2005, Georgia 2005.

marks alone.

This research uses a contest version of an all-pay auction with budget constraints to connect the literature on CCA with the literature on shadow education (a collection of fee-based institutions that fall outside formal schooling institutions and have been designed to enhance the students' formal school careers (Stevenson and Baker 1992)) and explains the experience of ex-Soviet republics. The model accounts for three empirical regularities: (1) the concentration of TP schools in higher income geographical areas; (2) the elevation of examination marks; and (3) the growing number of institutions looking for an alternative to CCA.

The model assumes that CCA examinations, which vary very little in their general designs year to year, enable the TP industry to accumulate unproductive test-specific knowledge (TSK), which is only used to allow private clients to achieve higher examination marks by complementing their skills.[2] Thus, while some activities in the TP industry are productive and increase applicants' skills, other activities systematically increase the examination marks of applicants who have more access to TP schools, not by increasing their skills but by providing them with TSK that complements their skills.

The model shows that if access to TSK is explicitly considered, applicants may be admitted to colleges who do not necessarily possess high levels of skills. Thus, examination marks have become less informative about applicants' underlying skills. If college wage premiums are high, then the gaming of CCA by those with more TP opportunities may decrease the quality of sorting under the CCA system. Applicants with more opportunities to access TSK may decide it is rational to invest more funds into TP schools (regardless of the productivity of TSK) to increase their chances against less fortunate applicants who are unable to access TSK.

[2]Examples of TSK are clearly evident from a typical advertisement of test preparation schools and include, for example, practice questions from previous exams or those that closely resemble them; learning non-obvious but efficient approaches to certain questions; and paying more attention to topics and vocabulary that are typical for the exam.

2 Motivation

2.1 College admission test-preparation

In recent years, the enormous growth of the college admission TP[3] industry and aggressive TP practices globally have perpetuated social inequalities, negatively affected the sorting properties of the public system of education, and led to resources being consumed that could be better used in other places. This has occurred in Asia, the Middle East, Europe, North America and Australia (Aurini et al. 2013; Bray 2011; Bray and Lykins 2012). In some countries, the TP industry has come to occupy a grotesquely large role in households. For example, in Turkey, Korea, Azerbaijan and Mauritius, students have stopped attending their public high schools en masse (or are using them as places in which to sleep) so that they can attend TP schools later in the day (Bhorkar and Bray 2018; Bray 2017). In other countries, household expenditure on TP schools represents a noticeable fraction of gross domestic product. Notably, the exorbitant financial burden that tutoring places on parents led to the TP industry being banned in Korea in 1980 (Bray 2009; Kim 2007). Today, the same appears to have occurred in Turkey. In general, the reasons for the development of the TP industry and the propensity of households to spend money on TP differ somewhat from country to country; however, "high-stake" college admission examinations are systematically implicated. This was particularly clear during recent introductions of CCA systems to ex-Soviet republics.

In the Soviet Union, and in other independent states that emerged after the Soviet Union's collapse in 1991, college specific examinations remained at the core of the CCA system. The system had its inefficiencies,[4] but demonstrated, after the collapse of the Soviet Union, a particular propensity towards corruption.[5] Those with higher incomes, connections and closer physical proximities

[3] The TP industry is a noticeable part of shadow education (Dang and Rogers 2008), but not the only one. One curious exception is Japanese Juku that distinctly involves students at the end of lower secondary school (Kimura 2019).

[4] Often, each department within each college has administered examinations independently. Many examinations have been delivered orally and can only be taken at the college where they are administered, essentially physically limiting access to education for some applicants (Drummond and Gabrscek 2012). Applicants also have to make college specific investments and take a new examination for each college to which they apply, and since they cannot do this at a single sitting, they must wait for a new test-taking occasion, which delays their entry by a year or more (Heyneman 2004).

[5] In Georgia and Kyrgyzstan, admission bribes became universal; both, however, had the same problem during the Soviet period. In Russia and the Ukraine, TP took an unusual fragmented form where each college has become a sole supplier of TP for its own admission test (Denisova-Schmidt and Leontyeva 2017; Silova and Bray 2006). This arrangement was, in fact, a semi-institutional bribe. Starodubtsev (2011) notes that in Russia by 2004 only 38% of applicants considered it corrupt to be admitted into college due to participation in

to colleges found themselves in privileged positions that allowed them to almost exclusively use the publicly-financed HE system.[6] CCA was introduced to increase the efficiency of the admission system and equalize the educational opportunities available to various demographic groups. Under the CCA system, any applicant knows the general examination structure, can sit the examination for free, and is able to broadcast their examination marks to a range of colleges.

The Ministry of Education and Science of Georgia provides unique insights into the practical implementation of the CCA system in Georgia (Gorgodze 2007).[7] The Georgian capital (which is the locality of most and all prestigious Georgian universities) allocates college degrees across the whole country; however, most of the country is rural. The introduction of CCA initially equalized admission opportunities for households located outside the capital; however, after only a few admission cycles, residents who lived in the capital discovered that, rather than paying semi-institutional bribes, they could invest money into TP schools. Due to the systematic increase in these students' marks, the eventual allocation of degrees is now much closer to what it was before the policy change.

Russia has had a remarkably similar experience. In Russia, the sequential introduction of CCA (from 2001 to 2009) can be used to show how the absence and presence of the TP industry has led to applicants being sorted differently. A report for the government by Efendiyev and Reshetnikov (2010) on the experimental introduction of CCA in several regions in 2001 (at a time at which TP schools were not yet common) showed an initial increase in the admission of students from less fortunate backgrounds. However, more than a decade later, Prakhov and Yudkevich (2017) showed that the engagement of TP schools by applicants with higher incomes has substantially and reliably increased their centralized examination marks, which naturally crowds out applicants with lower incomes.

CCA leads to the introduction of examinations, the designs of which are generally publicly known. Consequently, preparation for examination becomes a homogeneous service and leads to the creation of an organized TP industry. Over time, learning by doing allows the TP industry to accumulate knowledge as to how centralized examinations can be passed.[8] However, a substantial portion

TP courses, regardless of the quality of their preparation and their exam mark.

[6] Kuzminov (2012) notes that in the late 1980s about 75% of students in Moscow colleges were from out of town; in early 2000 this number was about 25%.

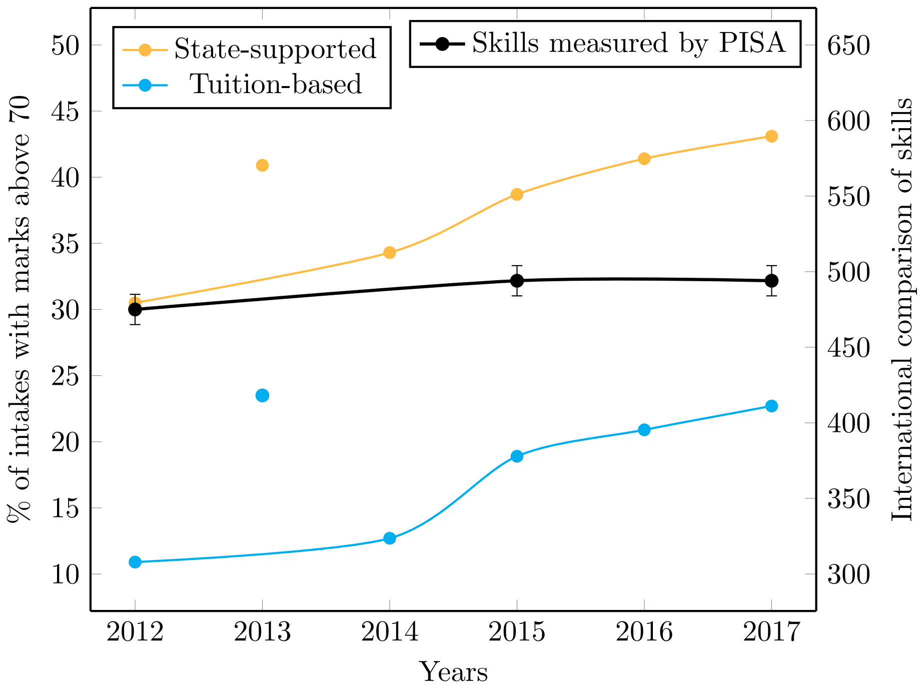
[7] I thank Mark Bray (the Director of Comparative Education Research Centre and UNESCO Chair Professor in Comparative Education at the University of Hong Kong) for sharing this article.

[8] A large body of empirical literature demonstrates *learning by doing*, or a positive asso-

of that knowledge is unproductive, as it does not increase applicants' skills and they cannot use it in their future studies. Thus, TSK creates a certain group of applicants that have systematically higher marks, but only because they know the examinations better. As marks become systematically less dependent on skills, a problem is created when colleges sort students based on their marks.

The above described regularity, as seen in the Georgian and Russian examples, likely applies to all standardized tests;[9] however, for this research, the introduction of CCA in Russia is considered as a case study. The following two facts summarize the key observations that accompanied the introduction of CCA in Russia.

Figure 1: Exam marks elevation unrelated to skills



Notes: Orange (tuition waived) and cyan (tuition not waived) lines show an increase in the fraction of admitted applicants with exceptionally high centralized exam marks (scoring more than 70 out of 100). The black line shows average skills in reading, mathematics and science of 15-year-old students measured by the Programme for International Student Assessment. Marks for 2013 are ignored because questions were leaked to the public right before the test day (Gushchin 2013).
Sources: M. S. Dobryakova (2017), OECD (2017)

ciation between current and past labor productivity. Furthermore, due to high competition, firms are unlikely to totally exclude outsiders from their stock of knowledge, which further fosters TSK accumulation (e.g., Lieberman 1984; Wright 1936).

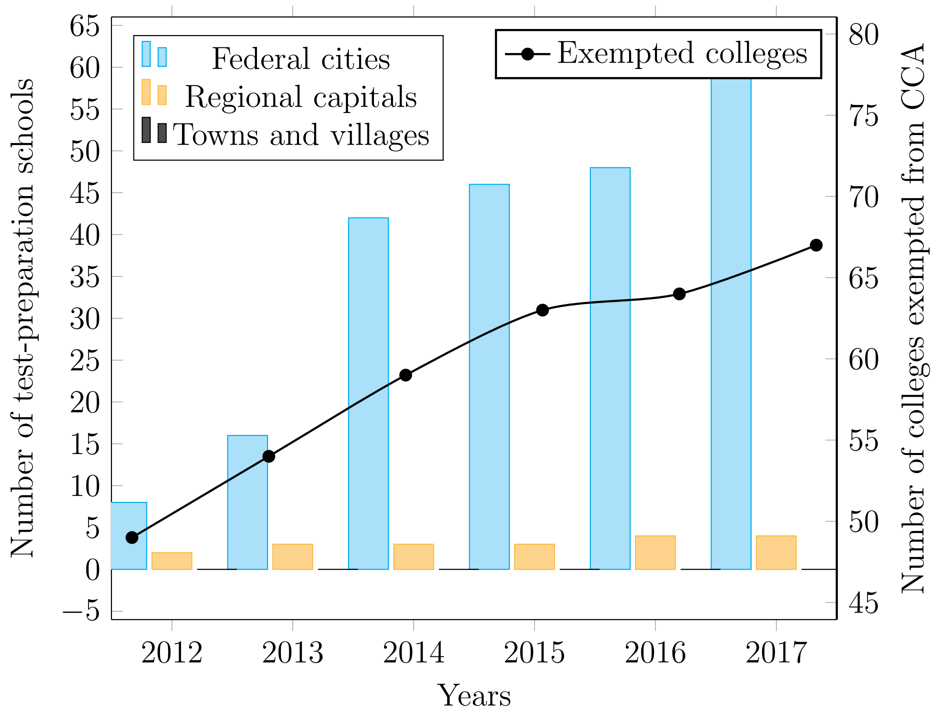
[9] A clean example is the Law School Admission Test, an admission test widely used in the United States that has created a large TP industry, which has been extensively criticized for its inability to predict applicants' performance as lawyers (Haddon and Post 2006).

Fact 1. *After CCA is introduced, the right tail of centralized examination marks distribution constantly increases. Concurrently, an alternative measure of skills indicates that the level of high school students' skills remains approximately the same (see Figure 1).*

The fraction of applicants receiving exceptionally high marks in the centralized examination constantly increases,[10] indicating that the aggregate capacity of applicants to do well in the centralized examination increases. However, as the increase is not driven by skills, it must be driven by another factor.

Fact 2. *The number of TP schools continues to increase and such schools tend to be more concentrated in higher income areas. Concurrently, the number of colleges that asked if they can use other tests (in addition to the centralized examination) has continued to strictly increase (see Figure 2).*

Figure 2: Growth of TP industry and CCA exemptions



Notes: Bars demonstrate that the growth of private preparation schools is concentrated in high income areas. The black line demonstrates the total number of colleges allowed to have complementary exams to sort applicants.
Sources: 4EGE (2017), YP (2017), Spark-interfax (2017)

[10] I am grateful to Elena Novikova (Higher School of Economics in Moscow) for providing the data. Note that marks go up for students who study for free *and* those who pay tuition; thus, the elevation is unlikely driven by a decrease in the number of available college places or number of government scholarships.

This suggests that the TP industry drives the elevation of the examination marks by systematically accumulating TSK and dispensing it to those applicants who can afford it. Consequently, it is more difficult for colleges to use centralized examination marks to sort applicants.[11]

2.2 Related game theoretical literature

Research on both CCA by economists and the sorting repercussions of aggressive TP practices by educational scientists is extensive. Thus, it is somewhat surprising that this paper appears to be the first to attempt to merge these two areas together.

An intuitive starting point for this study is Spence's (1973) famous model, under which the level of investment into TP is interpreted as a decision in a signaling game. College admission regulates the allocation of a scarce signal that gives college graduates access to better jobs and higher salaries. Thus, if a standardized test is used as a screening device, households have a strong incentive to invest in preparing for this test, regardless of the productivity of the knowledge to be gained.

Recently, Bodoh-Creed and Hickman (2018) employed an auction-theoretical treatment of college admission, where a continuum of students sought to enroll into a continuum of colleges, to undertake a policy analysis of the United States' college admission systems. Hafalir et al. (2018) also employed an auction-theoretical treatment of college admission to compare applicants' efforts in relation to CCA and decentralized college admission. Economically, the model suggested in this dissertation's research is similar to these models; technically, it is a mixture of Moldovanu and Sela's (2001) contest model and Che and Gale's (1996) budget constraints auction model.

The contest model is a variation of an all-pay auction that was analyzed in a series of works by Moldovanu and Sela (2006) and Moldovanu, Sela, and Shi (2012), who demonstrated an optimal combination of the prize and punishment that maximizes participants' efforts. Konrad (2009) conducted a rich survey of various applicants on all-pay auctions and contests. The best symmetric responses in auctions with budget constraints were originally analyzed by Che and Gale (1998, 2000), who ranked auction formats by revenue and showed the failure of revenue equivalence. Later, Kotowski and Li (2014) generalized Che

[11]Note that the preponderance of TP schools in larger cities does not preclude applicants from commuting there, or from buying an online course or a book. Further, an increase in the number of exempted colleges could be a reaction to an excessive deployment of CCA by the government in the first place.

and Gale's approach in an all-pay auction with budget constraints and affiliated values. The model in this paper adopts the same assumption as that adopted in the literature on auctions with budget constraints.

3 The Model

3.1 Agents and centralized exam marks

CCA exists to help colleges sort high school graduates according to their skills in a given college admission cycle. As skills are not observed and college applicants lack incentives to truthfully reveal their skills, a centralized examination, which is at the core of the CCA system, overcomes this problem by asking all applicants to sit the same examination so that the sorting can be based on observed examination marks. If the test-taking conditions were identical and nothing other than an applicant's skills were to inform examination marks, then an applicant with higher skills should achieve a higher mark, which would make the sorting outcome identical to one in which an applicant's skills were publicly observed. The assumption of the existence of a perfect proxy allows assortative outcomes to occur under CCA, as shown by Che and Koh (2016).

Test-taking conditions are not identical. There are noises of different origin that affect marks, which still makes sorting outcomes assortative, but only on average. Access to TSK is not a noise, but an inherent characteristic of applicants. To capture this idea, take two college applicants denoted by $\{s_i^\theta\}_{i=1}^2$ and defined as two $i.i.d$ privately known draws $\{(V_i, W_i)\}_{i=1}^2$ from a joint probability density function $f_{V,W}(v, w)$ defined over the applicants' potential characteristics: $\text{supp}(V) = [0, 1]$, $\text{supp}(W) = [\underline{w}, \overline{w}]$, and $0 \leq \underline{w} < 1 \leq \overline{w}$. The realized v represents the applicant's skill level and the realized w represents access to TSK (which is assumed to be accumulated by the TP industry).

To reduce the dimensionality of the private space, the model employs an assumption that was first proposed by Che and Gale (1996) and later gained wide acceptance (Krishna 2009, p. 42). However, instead of reading w as a cap on bidding, this study interprets observed marks $b(v, w)$ as an extreme form of complementarity between an applicant's skills, v, and their access to TSK, w.

Assumption 1. *An examination mark that conveys an applicant's skill level must be complemented by TSK. Further, the higher the desired mark, the higher the necessity of TSK:*

$$b(v, w) \overset{def}{=} \min\{b(v), w\}. \tag{1}$$

Economically, the assumption views the observed mark of an applicant as a complementary mixture of TSK, w, and $b(v)$. The latter is an examination mark that would have perfectly signaled the applicant's skill level (if TSK were assumed away). An excess of TSK will not necessarily increase an applicant's marks; similarly, being exceptionally skillful will not necessarily translate into higher marks in the absence of TSK. Thus, applicants could be highly skilled, but if they have no training on, for example, the questions typical to the examination, their marks may not reflect their skills. Further, a higher value of w is required to signal a higher v. For example, a highly-skilled applicant still has to do full-fledged practice tests in order to achieve a result that signals his/her skill level. Conversely, to achieve a moderate result, a moderately skilled applicant may only need to complete practice questions from free textbooks or online materials. Under this interpretation, marks are explicitly formulated as an isolated contribution of skills when TSK is controlled for, and as an isolated contribution of TSK when skills are controlled for, while the assumed perfect complementarity accentuates the importance of TSK. Finally, it should be noted that the TP industry can still positively effect v, as the assumption only accounts for the unproductive component of TP activity. Intuitively, it could be imagined that a typical TP school offers two services. The first service only helps students to read school textbooks (productive knowledge). The second service, which is of importance to this study, only trains students to pass the examination (unproductive knowledge).

It might seem that those deprived of TSK are doomed to have lower marks, regardless of their skills. This may not be an overstatement in light of the evidence available to educational scientists; however, it generally depends on equilibrium test-taking behaviors, which depend on other parameters.

Finally, two colleges are denoted as $\overline{c}$ and $\underline{c}$. It is public knowledge that graduating from college $\overline{c}$ gives a wage premium $\overline{\beta} \in \mathbb{R}_{>0}$, whereas $\underline{c}$ gives $\underline{\beta} \overset{\text{set}}{<} \overline{\beta}$. Thus, an applicant that is accepted into $\overline{c}$ will get benefit from an extra wage premium $\beta \overset{\text{def}}{=} \overline{\beta} - \underline{\beta}$. The premium of $\underline{c}$ is normalized $\underline{\beta} \overset{\text{set}}{=} 0$. In effect, analytically, colleges are reduced to a single real-valued parameter β, and below several interpretations to β are made.

If $\overline{c}$ and $\underline{c}$ are understood to be different colleges or even two groups of colleges (e.g., regional vs. federal), then the size of β captures the presence of a highly competitive college that provides graduates with access to highly paid jobs. This interpretation provides insight into why most competitive colleges (e.g., the top colleges in Russia or Japan) tend to prefer a decentralized admission processes.

Working with a two-by-two case enriches the model with two additional interpretations, which are less directly inspired by the above mentioned Georgian and Russian examples. If $\overline{c}$ and $\underline{c}$ are levels of education (e.g., a graduate school and college degree, or a high school diploma and a college degree), then β is an extra wage premium between levels of education and the model interprets applicants $\{s_i^\theta\}_{i=1}^2$ as two aggregated groups, one of which could be "unprivileged". This interpretation sheds light on cross-country differences in the application of CCA (e.g., in the United States).[12] Finally, if $\overline{c}$ and $\underline{c}$ are different majors, then the size of β captures the presence of unusually well-paid specializations. This interpretation helps to explain why some specializations prefer decentralized allocation while others prefer centralized allocation (e.g., the medical profession in Ireland or Australia).

In summary, the game is denoted by $\left(\left\{s_i^\theta, b_i(v_i, w_i)\right\}_{i=1}^2, \beta, f_{V,W}(v,w)\right)$ where:

$$
\begin{aligned}
s_i^\theta &- \text{an applicant } i \text{ with type } \theta \overset{\text{def}}{=} (v,w),\\
b_i(v_i,w_i) &- \text{applicant } i\text{'s centralized exam mark},\\
v_i &- \text{applicant } i\text{'s unobserved skills},\\
w_i &- \text{applicant } i\text{'s unobserved access to TSK},\\
\beta &- \text{the institution's extra wage premium},\\
V,W &- \text{random variable for skills and TSK, respectively.}
\end{aligned}
$$

3.2 Sequence of actions and the utilities functions

The basic rules of CCA are fairly well known. In a stylistic version of this system, both applicants simultaneously sit a centralized examination from which they receive their results in the form of examination marks $\{b_i\}_{i=1}^2$. The applicant with the highest mark is assigned to $\overline{c}$; the other applicant goes to $\underline{c}$. The following utility function captures this for an arbitrary applicant, i:

$$
\mathbb{U}_{s_i^\theta} = \begin{cases} \beta - {}^{b_i}/_{v_i} & \text{if admitted to } \overline{c} \\ -{}^{b_1}/_{v_i} & \text{if admitted to } \underline{c} \end{cases}. \tag{2}
$$

This utility function has been extensively exploited in the literature and originates from the work of Moldovanu and Sela (2001).

[12] Likewise, it provides an alternative explanation for the persistence of the decentralized functioning of the job market for new economists that is usually attributed to the absence of a pervasive market failure that is believed to generate a widespread interest in centralization (Coles et al. 2010). Having a good working paper does not guarantee getting an academic job, as it is not clear how much advice was given to a PhD student by their supervisor when writing their paper.

Assumption 1, while having the already discussed appealing economic content, offers a significant analytical convenience. One could think of $b(v)$ as the examination mark of an applicant who is "not constrained" in his/her access to TSK, but who understands that the competing applicant could be constrained, and so on (i.e., common knowledge assumption). Now, to derive a mutual best response, unconstraint types can be solely focused upon. This approach is consistent with previous studies by Che and Gale (1998), Fang and Parreiras (2002, 2003) and Kotowski (2010) and Kotowski and Li (2014).

Finally, to use auction-theoretical tools, the following is further assumed about $b(v)$.

Assumption 2. *Applicants with the same skills will achieve the same examination marks:*

$$b_1(\cdot) = b_2(\cdot) = b(\cdot). \tag{3}$$

Additionally, marks strictly increase with skills:

$$b(v_i) > b(v_j) \Leftrightarrow v_i > v_j \quad i, j \in \{1, 2\} \quad i \neq j. \tag{4}$$

Taken together, these two assumptions imply that Applicant 1 distinguishes three states:

$$\mathbb{U}_{s_1^{(v_1,\overline{w})}}\Big(b, b(\theta_2)\Big) = \begin{cases} \beta - {}^{b_1}/_{v_1} & \text{if } v_1 > v_2 \text{ and } w_2 > b(v_2) \text{ or } w_2 < b(v_2), \\ \beta - {}^{b_1}/_{v_1} & \text{if } v_1 < v_2 \text{ and } w_2 < b(v_2), \\ -{}^{b_1}/_{v_1} & \text{if } v_1 < v_2 \text{ and } w_2 > b(v_2) \end{cases}. \tag{5}$$

In the first state, Applicant 1 wins, as he/she is the most skillful regardless of Applicant 2's access to TSK. In the second state, Applicant 1 wins despite Applicant 2 being more skillful, but fails to complement his/her skills with TSK. In the third state, Applicant 1 loses because Applicant 2 is more skillful and has better access to TSK.

College $\overline{c}$ distinguishes two states:

$$\mathbb{U}_{\overline{c}} = \begin{cases} v_1 & \text{if } \min\{b(v_1), w_1\} \geq \min\{b(v_2), w_2\} \\ v_2 & \text{if } \min\{b(v_1), w_1\} \leq \min\{b(v_2), w_2\} \end{cases}. \tag{6}$$

These two outcomes can be achieved in four different ways (see Table 1). Taking the expectation over all possible states produces:

$$\mathbb{U}^e_{\overline{c}}(b(v, w)) = A + B + C + D \tag{7}$$

where

$$A = \int_0^1 \int_{b(v_2)}^{\overline{w}} \int_0^1 \int_{b(v_1)}^{\overline{w}} v_1 \mathbb{1}\{v_1 \geq v_2\} + v_2 \mathbb{1}\{v_1 \leq v_2\} f(v_1, w_1) f(v_2, w_2) dw_1 dv_1 dw_2 dv_2 \tag{8}$$

$$B = \int_0^1 \int_{\overline{w}}^{b(v_2)} \int_0^1 \int_{b(v_1)}^{\overline{w}} v_1 \mathbb{1}\{b(v_1) \geq w_2\} + v_2 \mathbb{1}\{b(v_1) \leq w_2\} f(v_1, w_1) f(v_2, w_2) dw_1 dv_1 dw_2 dv_2 \tag{9}$$

$$C = \int_0^1 \int_{b(v_2)}^{\overline{w}} \int_0^1 \int_{\overline{w}}^{b(v_1)} v_1 \mathbb{1}\{w_1 \geq b(v_2)\} + v_2 \mathbb{1}\{w_1 \leq b(v_2)\} f(v_1, w_1) f(v_2, w_2) dw_1 dv_1 dw_2 dv_2 \tag{10}$$

$$D = \int_0^1 \int_{\overline{w}}^{w_1} \int_0^1 \int_{w_2}^{\overline{w}} v_1 \mathbb{1}\{w_1 \geq w_2\} + v_2 \mathbb{1}\{w_1 \leq w_2\} f(v_1, w_1) f(v_2, w_2) dw_1 dv_1 dw_2 dv_2. \tag{11}$$

Table 1: Sample space of college's utility function

	$b(v_2) \leq w_2$		$b(v_2) \geq w_2$	
$b(v_1) \leq w_1$	A:	v_1 if $v_1 \geq v_2$ v_2 if $v_1 \leq v_2$	B:	v_1 if $b(v_1) \geq w_2$ v_2 if $b(v_1) \leq w_2$
$b(v_1) \geq w_1$	C:	v_1 if $w_1 \geq b(v_2)$ v_2 if $w_1 \leq b(v_2)$	D:	v_1 if $w_1 \geq w_2$ v_2 if $w_1 \leq w_2$

3.3 Equilibrium test-taking behavior

For heuristic derivation, the game is treated as a revelation mechanism. Both applicants report their types to a mediator who sits exams for them. Assume truth-telling by Applicant 2. Then, in a symmetric equilibrium, the expected utility for Applicant 1 of type $(v_1, \overline{w})$ with the mediator choosing $b(x)$ for him/her is:

$$
\begin{aligned}
\mathbb{U}^e_{s_1^{(v_1,\overline{w})}}(b(x)) &= \mathbb{E}_{\theta_2}\Big[\mathbb{U}_{s_1^{(v_1)}}\Big(b(x), b(\theta_2)\Big)\Big] \\
&= \beta\mathbb{P}[v_2 < x] + \beta\mathbb{P}[v_2 > x]\mathbb{P}[w_2 < b(x)] - \frac{b(x)}{v} \qquad (12)\\
&= \beta F_V(x) + \beta(1 - F_V(x))F_W(b(x)) - \frac{b(x)}{v}.
\end{aligned}
$$

The first line states that Applicant 1's utility, as prescribed by the Bayes Nash equilibrium, is an expected value over pay-off relevant states with respect to the competing types of applicants. The second line captures an admission when he/she is the most skillful and an admission when he/she is not skillful, but the competing applicant lacks TSK. The last line holds by the definition of the cumulative distribution function. The first order condition for truth-telling to be an equilibrium is that the derivative of $\mathbb{U}^e_{s_1^{(v_1,\overline{w})}}(\cdot)$ with respect to x evaluated at v is equal to zero. Namely, the condition

$$
\left.\frac{\partial \mathbb{U}^e_{s_1^{(v_1,\overline{w})}}(b(x))}{\partial x}\right|_{x=v} \overset{\text{set}}{=} 0 \qquad (13)
$$

takes the form of

$$
-\beta F_W(b(v))f_V(v) + \beta f_V(v) - \frac{b'(v)}{v} + \beta(1 - F_V(v))f_W(b(v))b'(v) \overset{\text{set}}{=} 0, \qquad (14)
$$

which can be simplified to identify a differential equation that characterizes $b(v;\beta)$, with a real valued parameter β denoted explicitly for expositional convenience. The following proposition summarizes the results.

Proposition 1. *Suppose the environment satisfies Assumptions 1 and 2, then there exists a symmetric equilibrium of the form* $b(v,w) = \min\{b(v), w\}$ *in increasing piecewise differentiable strategies*

$$
b^{EQ}(v,w;\beta) = \begin{cases} \hat{b}(v;\beta) & \text{if } v \le \tilde{v} \\ \min\{b(v;\beta), w\} & \text{if } v > \tilde{v} \end{cases}, \qquad (15)
$$

where

(a) $\tilde{v} \overset{def}{=} \inf\{v \in [0,1] : b(v;\beta) \ge \underline{w}\}$;

(b) $b(\cdot)$ is implied in

$$b'(v;\beta) = \frac{\beta v f_V(v)\Big(1 - F_W\big(b(v)\big)\Big)}{1 - \beta v f_W\big(b(v)\big)\big(1 - F_V(v)\big)}, \tag{16}$$

which has a boundary condition $b(\tilde{v}) = \underline{w}$;

(c) Case $v \leq \tilde{v}$ yields a differential equation $\beta v f_V(v) = b'(v;\beta)$ that gives an explicit

$$\hat{b}(v;\beta) = \beta \int_0^v t f_V(t)dt. \tag{17}$$

Proof. Note that multiplying (12) by v produces

$$\beta F_V(x)v + \beta F_W(b(x))(1 - F_V(x))v - b(x), \tag{18}$$

which is an expected value over pay-off relevant states in an all-pay-auction with types $(v \in [0,\beta]) \times (w \in [\underline{w}, \overline{w}])$. Properties of the maximizer $b(x)$ in a contest and a generic form of an all-pay-auction are the same, due to homomorphism of the optimization problems. Therefore, the proposition is a special case of Theorem 1 in Kotowski and Li (2014), with two players and independent private values. □

The differential equation (16) is defined over a set $\{v \in [0,1] : b(v;\beta) \geq \underline{w}\}$ and it accounts for the change in marginal incentives faced by Applicant 1. A slight increase in investment in marks not only allows an applicant to outperform other applicants with slightly higher skill levels, v, but also to outperform applicants with sufficiently low access to TSK, w, regardless of their skill level, v.

A property of the differential equation is that it permits an explicit solution when $b(v;\beta) < \underline{w}$. The technical reason for this is that the value of $b(v;\beta)$ of this size is outside of the support of W, which makes $f_W(\cdot) = F_W(\cdot) = 0$. The solution to the differential equation under these conditions coincides with a one prize, two players tournament absent the budget constraint defined in Proposition 1 in Moldovanu and Sela (2001), and denoted here by $\hat{b}(v;\beta)$. The economic meaning is that an applicant with low skills will not strategize against applicants who might have less fortunate TP opportunities.

Example 1. *Assume* $V \overset{i.i.d.}{\sim} \mathcal{U}[0,1]$, $W \overset{i.i.d.}{\sim} \mathcal{U}[0.5, 2.08]$ *and* $\beta = 3$ *then*

$$b^{EQ}(v, w; 3) = \begin{cases} \hat{b}(v; 3) & \text{if } v \leq 0.57 \\ \min\{b(v; 3), w\} & \text{if } v > 0.57 \end{cases}, \tag{19}$$

where

$$\hat{b}(v; 3) = (3/2)v^2 \tag{20}$$

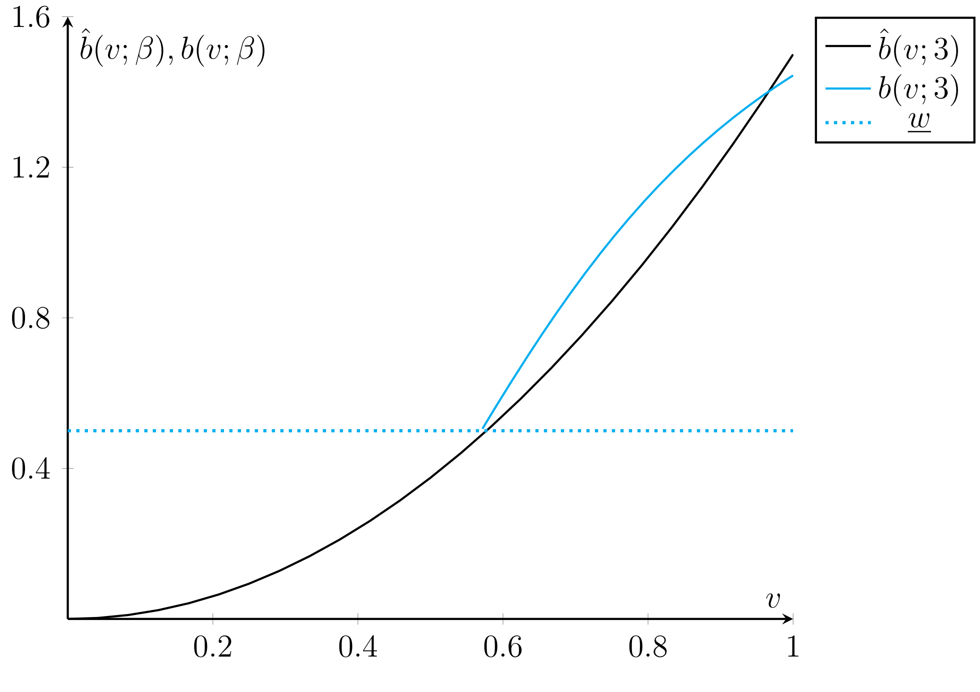
and $b(v; 3)$ *is implied in*

$$b'(v; 3) = \frac{3v\left(1 - \left(\frac{b(v) - 0.3}{2.08 - 0.5}\right)\right)}{1 - 3v\left(\frac{1 - v}{2.08 - 0.5}\right)}, \tag{21}$$

which has a boundary condition $b(0.57) = 0.5$.

$\hat{b}(v; 3)$ in Figure 3 represents the mark that an applicant with skill level, v, would have had if TSK were assumed away. In an explicit formulation of TSK, applicants with skill levels of $v < 0.57$ will still receive the same mark; however, applicants with skill levels of $v > 0.57$ will choose to invest more in preparation and will be awarded marks in the size of $b(v; 3)$. The parameter $\underline{w}$ plays a defining role here; in the case when $\underline{w} \geq b(1; \beta)$, function $b(v; \beta)$ never emerges and applicants of all types behave in accordance with $\hat{b}(v; \beta)$. Therefore, this model is a generalization of Moldovanu and Sela's (2001) model, as for a certain

Figure 3: Functions $\hat{b}(v)$ and $b(v)$ from Example 1.



parametrization the models become similar.

The model clearly indicates that under CCA certain types of applicants with better access to TSK will rationalize their overinvestment in TP not because they are concerned about their knowledge productivity, but merely to gain an advantage over those who may have less access to TSK.

4 Implications for Centralized College Admission

This section explores a change in the equilibrium behaviors and sorting outcomes in response to a change in the college wage premium, β, and a change in distribution, F_W.

A technical prelude is required. It is known from analyses of the war of attrition (Kotowski and Li 2014) and first (Che and Gale 1998) and second (Fang and Parreiras 2002) price auctions that the presence of a budget constraint may encourage more aggressive bidding. Due to the competing effects of budget constraints, behavior in all-pay auctions can go either way (Kotowski and Li 2014). A bidder is more optimistic regarding an item's value because, in equilibrium, that bidder may defeat an opponent with a high private value who has a low budget. However, budgets also stratify competition, as when fewer bidders are capable of competing at higher levels, the marginal incentive to bid higher declines. This serves to depress bidding.

A unique feature of a contest version of an all-pay auction is that β can regulate which effect will eventually dominate. For lower values of β, a budget constraint might depress bidding for some types (see Figure 3); however, for higher values of β, all types of bidders bid more aggressively.

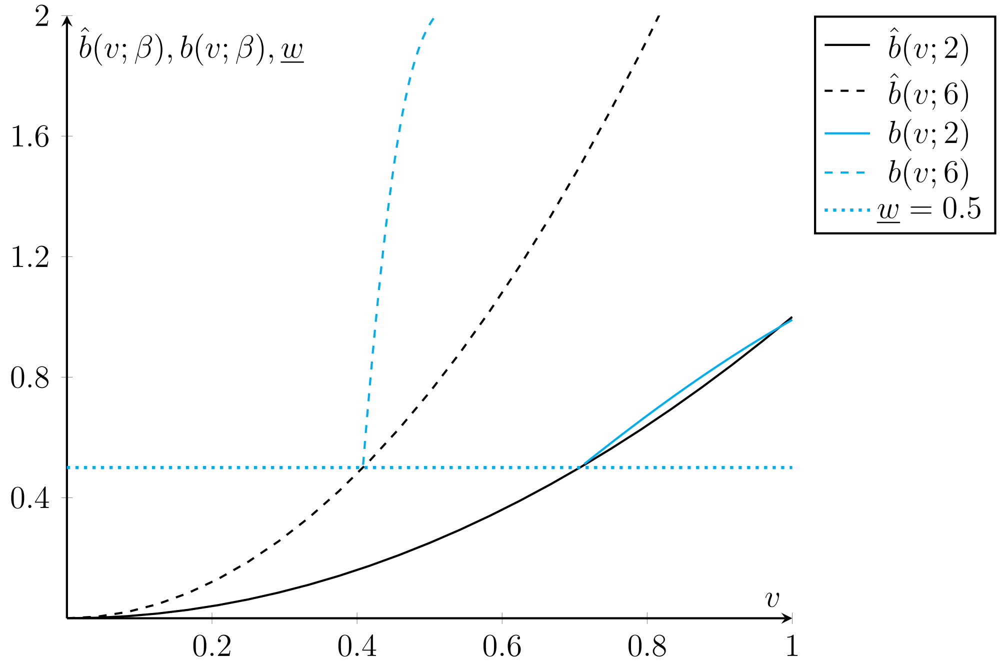
Some of this intuition carries over to a change in F_W: when an applicant is more likely to win admission into college because his/her opponent has a lower budget rather than a lower skill level. In the war of attrition, and first and second price auctions, the change encourages bidding. In an all-pay auction, the effect is ambiguous. These behavior features have economic significance for the college admission problem.

4.1 Comparative statics in wage premium

A change in β captures the change in equilibrium test-taking behaviors in relation to different colleges (groups of colleges, levels of education or specializations). Intuitively, a higher wage premium justifies a higher investment into activities that will lead to higher marks being achieved; however, this requires both high levels of skills and TSK.

Figure 4 uses Example 1 to show how applicants respond to a change in β. Prima facie intuition is indeed confirmed. The most prestigious colleges are overwhelmed by applicants with exceptionally high examination marks but mediocre skill levels. Accounting for TSK reveals that high wage premium colleges are more likely to be affected by gaming against applicants with fewer preparation opportunities. These results hold generally.

Figure 4: Comparative statics in β using Example 1.



Proposition 2. *Suppose $\beta' > \beta$, i.e., the college wage premium is higher. Then under the conditions of Proposition 1:*

(a) $\lim_{v \to \tilde{v}^+} b'(v;\beta) > \lim_{v \to \tilde{v}^-} b'(v;\beta)$;

(b) $\lim_{v \to \tilde{v}^+} b'(v;\beta') - \lim_{v \to \tilde{v}^-} b'(v;\beta') > \lim_{v \to \tilde{v}^+} b'(v;\beta) - \lim_{v \to \tilde{v}^-} b'(v;\beta) > 0$;

(c) $b(v;\beta') > b(v;\beta) > \hat{b}(v;\beta)$ *for* $\beta > \frac{F_W(b(v;\beta))}{f_W(b(v;\beta))} \frac{1}{1-F_V(v)} \frac{1}{v}$.

Proof. A direct calculation gives (a)

$$\begin{aligned}
\lim_{v\to\tilde{v}^+} b'(v;\beta) &= \lim_{v\to\tilde{v}^+} \frac{\beta v f_V(v)(1-F_W(b(v)))}{1-\beta v f_W(b(v))(1-F_V(v))} \\
&= \frac{\beta\tilde{v} f_V(\tilde{v})(1-F_W(b(\tilde{v})))}{1-\beta\tilde{v} f_W(b(\tilde{v}))(1-F_V(\tilde{v}))} \\
&= \frac{\beta\tilde{v} f_V(\tilde{v})(1-F_W(\underline{w}))}{1-\beta\tilde{v} f_W(\underline{w})(1-F_V(\tilde{v}))} \\
&= \frac{\beta\tilde{v} f_V(\tilde{v})}{1-\beta\tilde{v} f_W(\underline{w})(1-F_V(\tilde{v}))} \\
&= \frac{1}{1-\underbrace{\beta\tilde{v} f_W(\underline{w})(1-F_V(\tilde{v}))}_{<1}}\beta\tilde{v} f_V(\tilde{v}) \\
&> \beta\tilde{v} f_V(\tilde{v}) = \lim_{v\to\tilde{v}^-} b'(v;\beta).
\end{aligned} \tag{22}$$

The third line uses a boundary condition $b(\tilde{v}) = \underline{w}$, whereas the last line uses a differential equation for case $v \le \tilde{v}$.

To see (b), note that the above showed

$$\lim_{v\to\tilde{v}^+} b'(v;\beta) = \psi(\beta) \lim_{v\to\tilde{v}^-} b'(v;\beta) \tag{23}$$

where

$$\psi(\beta) \stackrel{\text{def}}{=} \frac{1}{1-\beta\tilde{v} f_W(\underline{w})(1-F_V(\tilde{v}))} > 1. \tag{24}$$

Clearly, $\psi'(\beta) > 0$ as

$$\frac{\partial}{\partial\beta}\left(\frac{1}{1-\beta\tilde{v} f_W(\underline{w})(1-F_V(\tilde{v}))}\right) = \frac{f_W(\underline{w})(1-F_V(\tilde{v}))\tilde{v}}{((F_V(\tilde{v})f_W(\underline{w}) - f_W(\underline{w}))\tilde{v}\beta + 1)^2} > 0. \tag{25}$$

An increase in β reinforces the gap between $b(\cdot)$ and $\hat{b}(\cdot)$.

A direct calculation gives (c)

$$\begin{aligned}
b'(v;\beta) &> \hat{b}'(v;\beta) \\
\frac{\beta v f_V(v)(1-F_W(b(v)))}{1-\beta v f_W(b(v))(1-F_V(v))} &> \beta v f_V(v) \\
\beta &> \frac{F_W(b(v;\beta))}{f_W(b(v;\beta))}\frac{1}{1-F_V(v)}\frac{1}{v}.
\end{aligned} \tag{26}$$

□

Thus, Part (a) of Proposition 2 shows that TSK always encourages more aggressive test-taking behaviors among some applicants (due to the realization of their relative advantage). Part (b) shows that an increase in the wage premium always makes test-taking behaviors more aggressive. Part (c) shows that, for a sufficiently high wage premium, applicants of all skill levels will behave more aggressively.

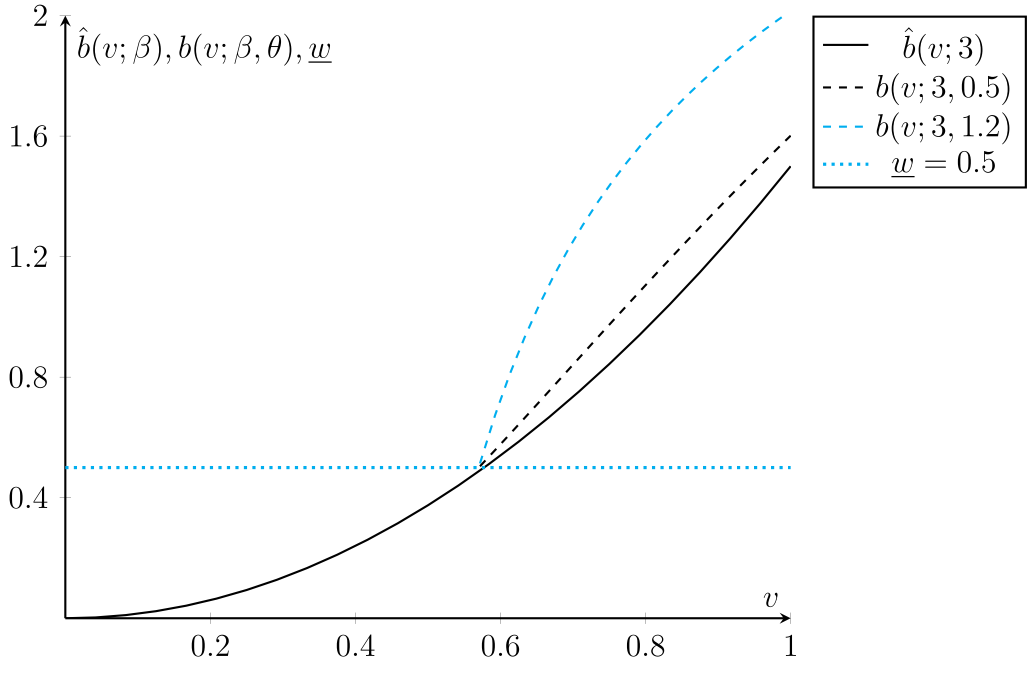
4.2 Change in access to TSK

As Figure 2 demonstrates, TP schools tend to be located in higher income areas. This tendency produces areas where applicants are more deprived of TSK in comparison to applicants from higher income areas who are less deprived of TSK. Such circumstances systematically place applicants from lower income areas in less advantageous positions. This mechanism can be studied in the current model as an increase in probability that one's opponent will fail to broadcast his/her skill level due to a lack of TSK.

It is natural to expect more aggressive test-taking behaviors ceteris paribus whenever a competing applicant is more likely to be constrained by TSK. CCA intensifies competition for admission into colleges, as a remote location stops being a barrier to become an applicant for a college sit. In this context, a sure way to game the system is to exploit non-universal access to TSK.

Consider Example 1, and replace distribution of TSK with $F_W(w;\theta) = 1 - \exp(-\theta(w - \underline{w}))$ distributed on $[\underline{w}, \infty)$ and fix $\underline{w} = 0.5$. An increase in θ makes one's opponent more likely to be constrained. Figure 5 shows equilibrium test-

Figure 5: Comparative statics in θ using Example 1.



taking behavior as the function of θ.

Proposition 3. *Suppose $F_{W'}(w)$ dominates $F_W(w)$ in terms of the reverse hazard rate, i.e., the likelihood that one's opponent would fail to complement his/her skills with TSK is higher. Then, under the conditions of Proposition 1:*

$$b_{W'}(v) > b_W(v) > \hat{b}(v) \quad for \quad \beta > \frac{F_W(b(v;\beta))}{f_W(b(v;\beta))}\frac{1}{1-F_V(v)}\frac{1}{v}. \tag{27}$$

Proof. The proof is similar to that in Krishna (2009, p. 47). Let $b > b_{W'}(\tilde{v}) = b_W(\tilde{v})$. Then $F_{W'}(w) < F_W(w)$ and $f_{W'}(w) > f_W(w)$. Thus, for all b where Equation (16) is positive:

$$0 < \frac{\beta v f_V(v)(1-F_W(b))}{1-\beta v f_W(b)(1-F_V(v))} < \frac{\beta v f_V(v)(1-F_{W'}(b))}{1-\beta v f_{W'}(b)(1-F_V(v))}. \tag{28}$$

If $b_{W'}(v)$ and $b_W(v)$ intersect, the former is steeper than the latter and this implies that they intersect once at most. Additionally, $b_W(v)$ would cross $b_{W'}(v)$ from above.

Thus, if $b_W(v) > b_{W'}(v)$ for any $v > \tilde{v}$, then $b_W(v) > b_{W'}(v)$ for $v \in (\tilde{v}, z)$, where z sufficiently small. Fix $\tilde{v} < v < z$ then

$$f_{W'}(b_{W'}(v)) \geq f_W(b_{W'}(v)) \geq f_W(b_W(v)) \tag{29}$$

and

$$F_{W'}(b_{W'}(v)) \leq F_W(b_{W'}(v)) \leq F_W(b_W(v)). \tag{30}$$

Therefore, $b'_{W'}(v) \geq b'_W(v)$.

Finally, because $b_W(v)$ and $b_{W'}(v)$ can be expressed as integral equation,

$$b_W(\tilde{v}) = b_W(z) - \int_{\tilde{v}}^{z} b'_W(x)dx > b_{W'}(z) - \int_{\tilde{v}}^{z} b'_{W'}(x)dx = b_{W'}(\tilde{v}) \tag{31}$$

which contradicts $b_{W'}(\tilde{v}) = b_W(\tilde{v})$. Therefore, $b_{W'}(v) > b_W(v)$. □

Thus, a better prospect for defeating an applicant with no access to TSK rationalizes more aggressive test-taking behaviors across applicants of all skill levels. Proposition 3 connects the concentration of TP schools in higher income areas, as shown by Figure 2, and the elevation of exams marks, as shown by Figure 1.

Importantly, there is another subtle economic counterpart to the comparative statics of Proposition 3, which reveals the unearned power that the TP industry possesses due to the presence of a certain information asymmetry. A

Bayes Nash equilibrium requires that a characterization of a bivariate random variable (V, W) be commonly known. It is plausible to envision that applicants would be able to make a sound guess on the characterization of V, due to the inherent long-term stability of the distribution of skills; however, it is somewhat less appealing to envision that applicants would be able to make a sound guess on the characterization of W. For example, an average applicant might know how skilled in math a competing applicant is, but is likely clueless as to how skilled the competing applicant is in sitting standardized exams that have been recently introduced. The support and the distribution of availability of TSK are unlikely to be stable long enough to become common knowledge. Thus, applicants at a given admission cycle should infer how many applicants do not use the TP industry to access TSK (e.g., come from areas that do not have a well-developed TP industry.).

In a typical advertisement campaign, a typical TP school will contend that its paying clients will gain access to an exclusive bank of questions that mimics the real test. Such TP schools are effectively declaring that they can help applicants who attend their school to outperform those who will not have access to such questions. The moment that parents accept this declaration, the mechanics of Proposition 3 apply.

Therefore, to explain the elevation of exam marks in Figure 1, there is no need for a massive TP industry to emerge in higher income areas, as in Figure 2. All it takes is a noisy advertisement campaign by a few TP schools, through which parents would attempt to infer the distribution of the availability of TSK and rationally respond by over-engaging the TP industry.

What is quite remarkable is that this outcome is entirely in line with an idea of normalization of aggressive TP practices, which has been extensively documented by educational scientists. Once aggressive TP practices enter a society, it is impossible to remove them (Bray 2009). The model exposes the force (i.e., information) that makes aggressive TP practices so persistent.

4.3 Influence on college's utility

Thus far, the model has shown that an increase in wage premium and the probability that a competing applicant lacks TSK intensifies test-taking behaviors. The following summarizes the consequences of such behaviors on the quality of sorting.

Theorem 1. *Suppose* $\beta' > \beta > \frac{F_W(b(v;\beta))}{f_W(b(v;\beta))} \frac{1}{1-F_V(v)} \frac{1}{v}$ *or* $F_{W'}(w) \leq F_W(w)$*, then* *under the conditions of Proposition 1:*

$$\mathbb{U}^{e}_{\bar{c}}(b(v,w);\beta') < \mathbb{U}^{e}_{\bar{c}}(b(v,w);\beta) \tag{32}$$

or

$$\mathbb{U}^{e}_{\bar{c}}(b_{W'}(v,w)) < \mathbb{U}^{e}_{\bar{c}}(b_{W}(v,w)). \tag{33}$$

Proof. Note that the results in Proposition 2 and Proposition 3 show that the conditions of this theorem guarantee that all types of applicants will behave more aggressively; that is, function $b(v)$ will attain a higher value for all v.

Now we inspect the definition of college utility $\mathbb{U}^{e}_{\bar{c}}(b(v,w))$ from Equation (7) to see what this aggression implies for the sorting quality.

Clearly, the first component of the utility function (event A) is strictly decreasing in b, since the area of integration is strictly smaller. The last component (event D) is independent of b and can be ignored.

Note that event B can be rewritten to remove $b(\cdot)$ from inside the integral:

$$\begin{aligned} B = &\int_{0}^{1}\int_{\underline{w}}^{b(v_2)}\int_{0}^{1}\int_{b(v_1)}^{\overline{w}} v_1 \mathbb{1}\{v_1 \geq b^{-1}(w_2)\} \\ &\quad + v_2 \mathbb{1}\{v_1 \leq b^{-1}(w_2)\} f(v_1,w_1) f(v_2,w_2) dw_1 dv_1 dw_2 dv_2 \\ = &\int_{0}^{1}\int_{\underline{w}}^{b(v_2)}\int_{0}^{1}\int_{b(v_1)}^{\overline{w}} v_1 \mathbb{1}\{v_1 \geq v_2\} \\ &\quad + v_2 \mathbb{1}\{v_1 \leq v_2\} f(v_1,w_1) f(v_2,w_2) dw_1 dv_1 dw_2 dv_2. \end{aligned} \tag{34}$$

An increase in the area of integration in w_2 is compensated by a decrease in the area of integration in w_1, and, since w is *i.i.d.*, event B does not change the utility. Event C is symmetric to event B; thus, the same reasoning can be applied to event C.

In sum, the utility is lower due to the influence of event A, whereas events B, C and D do not contribute to the change of utility. □

Proposition 2 and Proposition 3 show what could drive the skills-irrelevant elevation of exam marks, while Theorem 1 shows that the same forces explain the increasing number of colleges trying to exit CCA. Facts 1 and 2 are explained.

5 Conclusion

Inspired by wide ranging literature on shadow education, this paper uses auction-theoretical modeling to demonstrate a flaw in the college admission market design literature. For a clean illustration of this flaw, the paper explores a recent wave of introduction of CCA in ex-Soviet republics. It is shown that a disproportional accumulation of TSK (proxied by the emergence of TP schools in higher income areas; depicted in Figure 1) rationalizes more aggressive TP practices (as seen by the skills-independent elevation of marks; depicted in Figure 2). This makes the sorting of applicants troublesome for colleges, especially for those that are highly competitive, and incentivizes colleges to exit the CCA system (depicted in Figure 2). The key economic message is that the functionality of CCA eventually depends on students' access to a stock of TSK, which is created and managed by the TP industry and cannot be directly controlled or even credibly measured by a government regulator. The TP industry is an integral element of CCA. It benefits from an increase in the examination marks, but ignores the role of those marks in resolving uncertainty.

Conceptually, this flaw is not unique to the college admission context. In hunter-gatherer societies, a man would signal his devotion to a woman by delivering to her, as a present, a piece of meat from a hunt. In being delivered as a present, the piece of meat contains information about the man's skills as a hunter and how devoted he is to the woman (Allen et al. 2011; Hawkes and Bird 2005). However, today, if a present (such as an item of jewelry) were to be given, a modern woman would be unable to determine if the gift were given as a sign of the man's genuine devotion or simply because the man had passed a gift store on his way to meet her for their date and had sufficient money to buy the gift. Similarly, colleges cannot tell if applicants' high marks are driven by their skills or by their having greater access to TP opportunities.

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